WELLCOME TO MY PRESENTATION

Presented By

Ariful Islam

Work Shop Superintendent

(Tech/Mechanical)

Mechanical Technology

Mymensingh Polytechnic Institute

Subject : Strength of Materials

Sub:Code:67064

Chapter -06

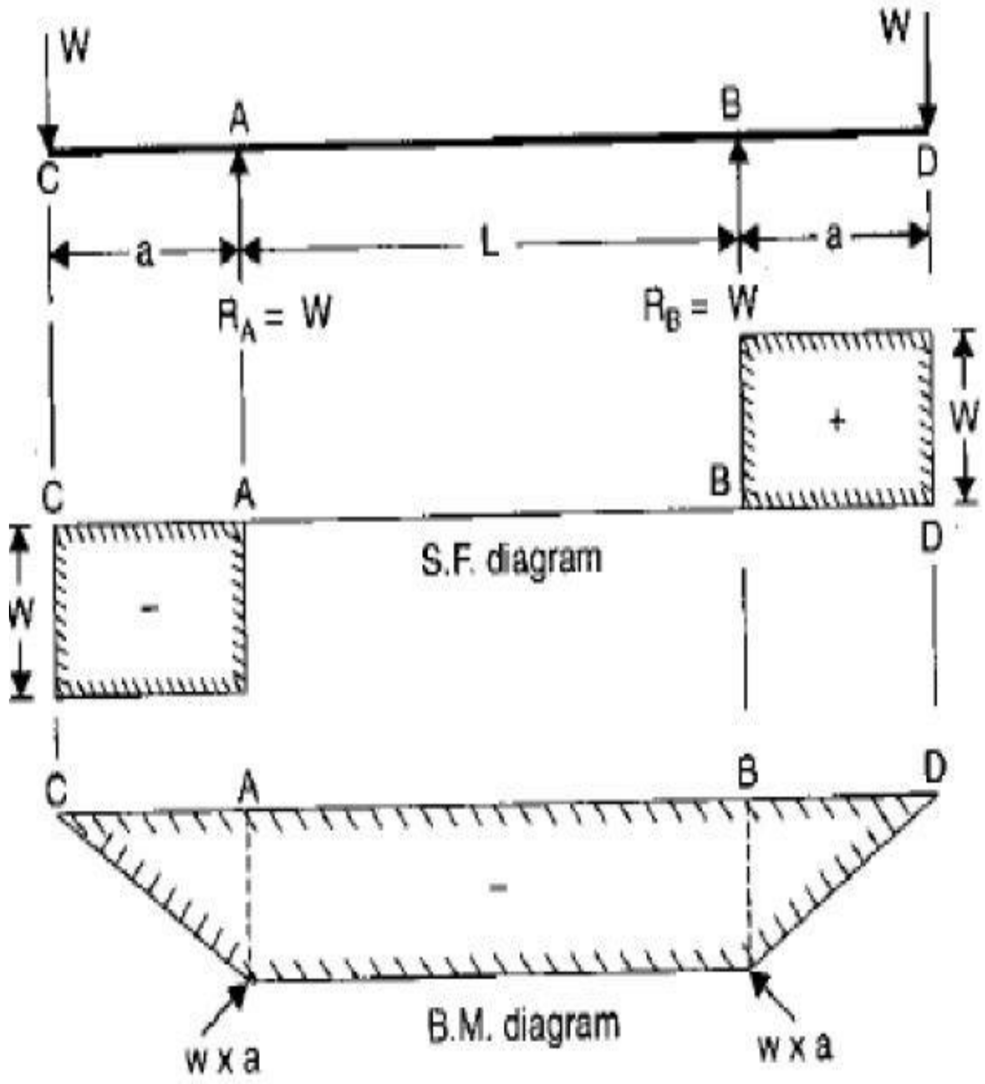
- Understand the analysis of bending stresses in beams.

After the end of this lesson students will able to;

- State the theory of simple bending.
- Explain bending stresses.
- Identify position of neutral axis.
- Define moment of resistance.
- Define section modulus.
- Solve problems related to bending stresses in beam.

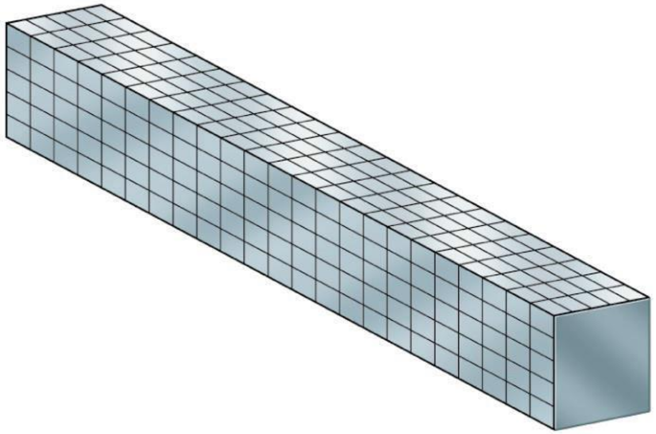
Stresses in Beams – Bending and Shear

- If a specific length of a beam is subjected to a constant bending moment & shear force is zero, then the stresses set up in that length of the beam are known as bending stresses. The length of the beam under constant bending moment is said to be in ***pure bending.***

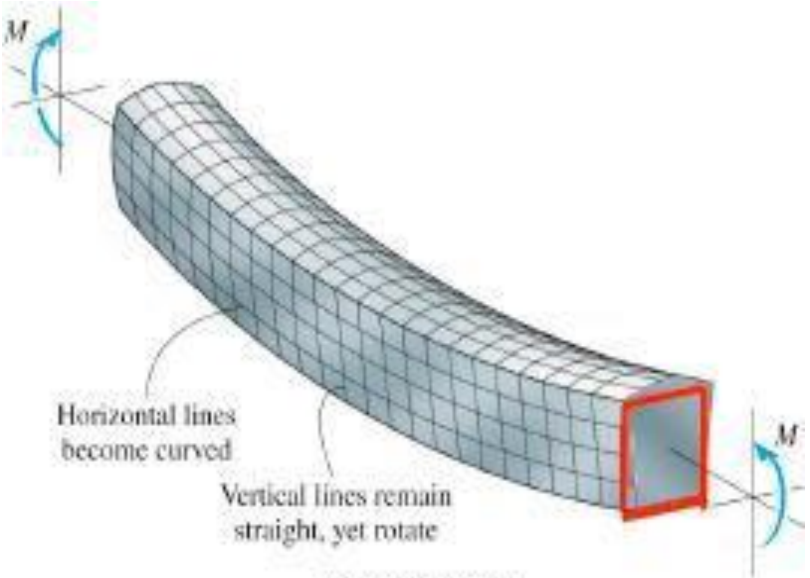


Stresses in Beams – Bending and Shear

- Internal bending moment causes beam to deform.
- Top fibers in compression, bottom in tension.
- **Neutral surface** – no change in length.
- **Neutral Axis** – Line of intersection of neutral surface with the transverse section.
- All cross-sections remain plane and perpendicular to longitudinal axis.



Before deformation



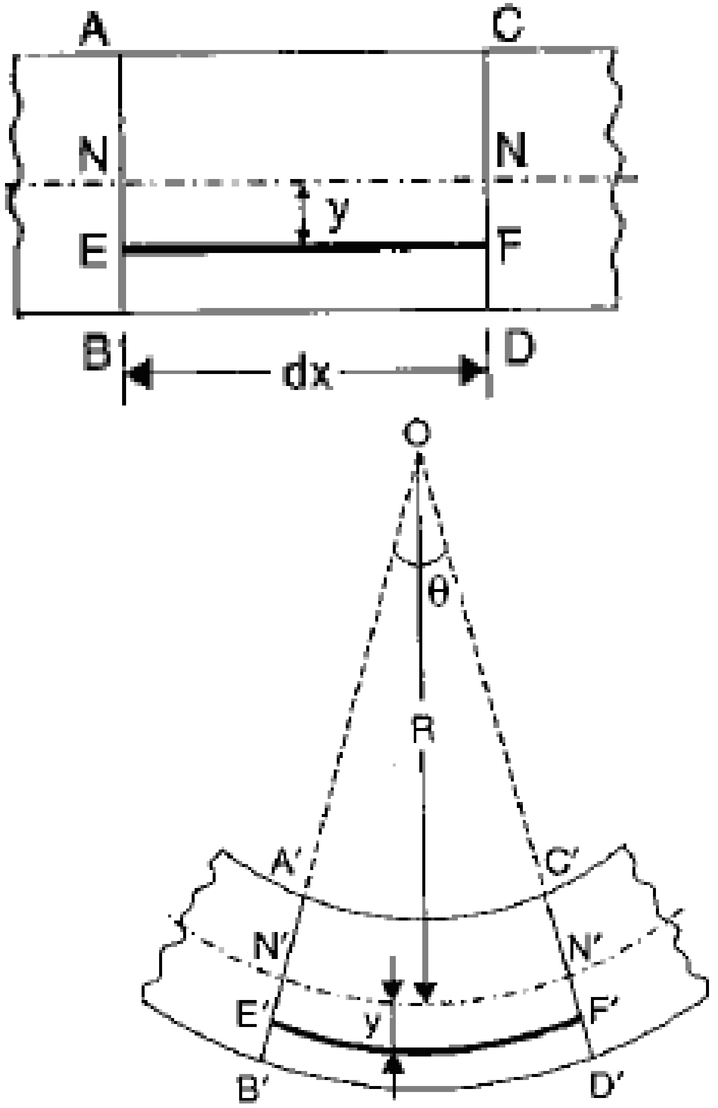
After deformation

Stresses in Beams – Bending and Shear

- Assumptions in simple (pure) bending theory:
  - Material of beam is homogenous and isotropic (same composition & constant E in all directions).
  - Young’s modulus is constant in compression and tension.
  - Transverse section which are plane before bending remain plain after bending (Eliminate effects of strains in other direction).
  - Beam is initially straight and all longitudinal filaments bend in circular arcs.
  - Radius of curvature is large compared with dimension of cross sections.
  - Each layer of the beam is free to expand or contract.

Derivation of Relationship Between Bending Stress and Radius of Curvature

- Consider a small length δx of a beam subjected to a simple bending as shown in the figure (a).
- Due to action of bending, the length δx will be deformed as shown in the figure (b).



Derivation of Relationship Between Bending Stress and Radius of Curvature

- Due to the decrease in length of the layers above N-N, these layers will be subjected to compressive stresses.
- Due to the increase in length of the layers above N-N, these layers will be subjected to tensile stresses.
- The amount by which a layer increases or decreases in length, depends upon the position of the layer w.r.t. N-N. This theory of bending is known as theory of simple bending.

Derivation of Relationship Between Bending Stress and Radius of Curvature

Let, R = Radius of curvature of neutral layer N'-N'.

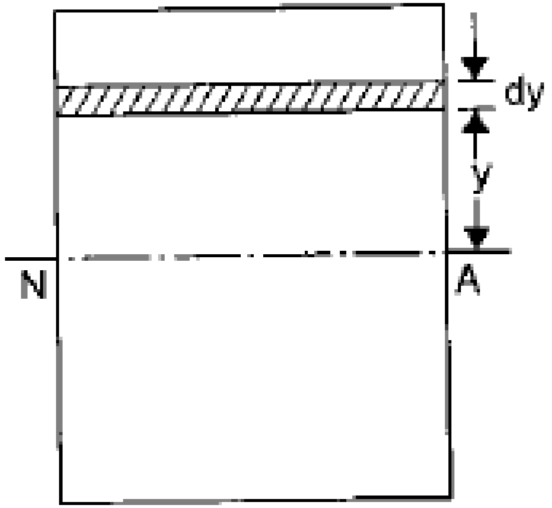
- θ = Angle subjected at O by A'B' and C'D' produced.
- y = Distance from the neutral layer.
- Original length of the layer = EF = δx = NN = N'N'
- From figure (b), N'N' = **R θ**
- Change (Increase) in length of the EF = E'F' –EF = **(R + y) θ – R θ**

  = y θ
- Strain in the layer EF = Increase in length / original length

  = **y θ / R θ = y / R**
- According to linear elasticity, σ α ϵ,.That is, ϵ = σ / E.
- $$\frac{y}{R} = \frac{\sigma}{E}$$ $$\sigma = \frac{E}{R} y$$ σ α y

Derivation of Relationship Between Bending Stress and Radius of Curvature (Moment of Resistance of a Section)

- The stresses induced in the layers of the beam create compressive and tensile forces.
- These forces will have moment about NA.
- The total moment of these forces about NA for a section is known as moment of resistance of that section.
- Consider a cross section of a beam as shown:



Derivation of Relationship Between Bending Stress and Radius of Curvature (Moment of Resistance of a Section)

Force on layer $= \dfrac{E}{R} \times y \times dA$

Moment of this force about N.A.

$$= \text{Force on layer} \times y$$

$$= \frac{E}{R} \times y \times dA \times y$$

$$= \frac{E}{R} \times y^2 \times dA$$

Total moment of the forces on the section of the beam (or moment of resistance)

$\therefore$ $$M = \int \frac{E}{R} \times y^2 \times dA = \frac{E}{R} \int y^2 \times dA$$

$\therefore$ $$M = \frac{E}{R} \times I \quad \text{or} \quad \frac{M}{I} = \frac{E}{R}$$

$\therefore$ $$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Euler – Bernoulli Bending Equation

Section Modulus (Z)

- It is the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis.

$$Z = \frac{I}{y_{max}}$$

I = Moment of inertia about neutral axis.

y_{max} = Distance of the outermost layer from the neutral axis.

Hence, $M = \sigma_{max} . Z$

- Thus, moment of resistance offered by the section is maximum when Z is maximum. Hence, Z represents the strength of the section.

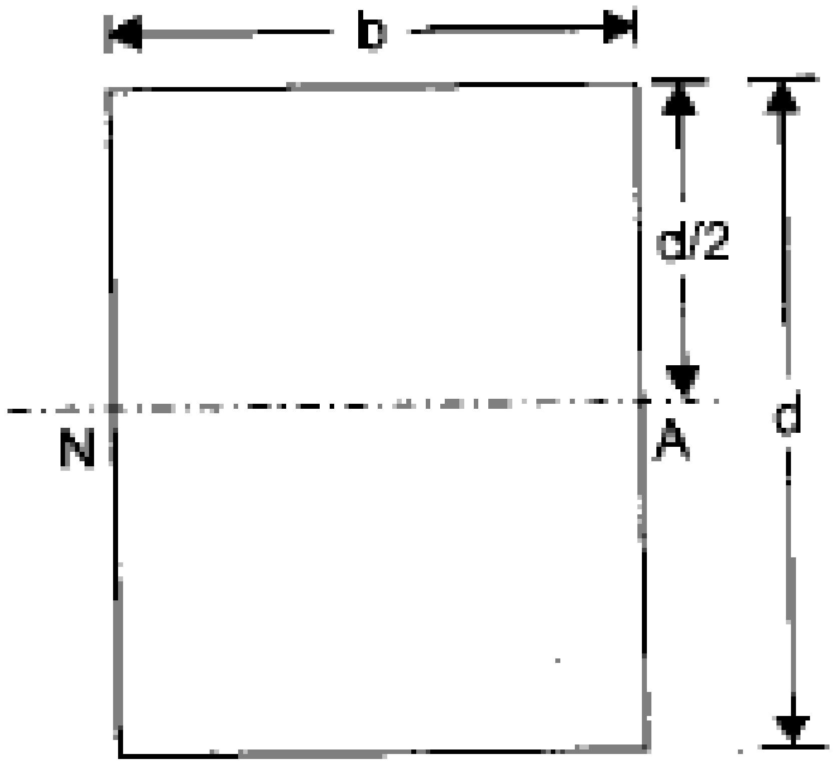
Section Modulus (Z)

1. Rectangular Section

$$I = \frac{bd^3}{12}$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{bd^2}{6}$$



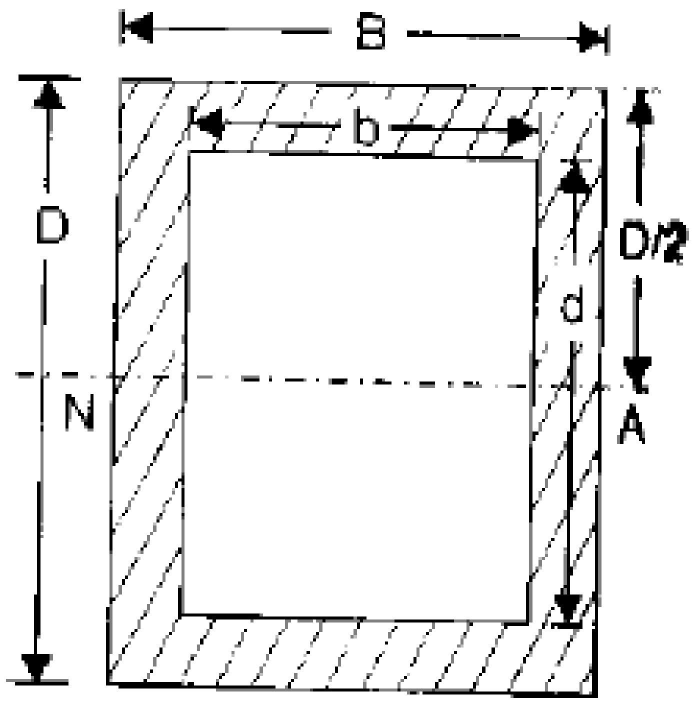
Section Modulus (Z)

2. Rectangular Hollow Section

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$y_{max} = \left(\frac{D}{2}\right)$$

$$Z = \frac{1}{6D}\left[BD^3 - bd^3\right]$$



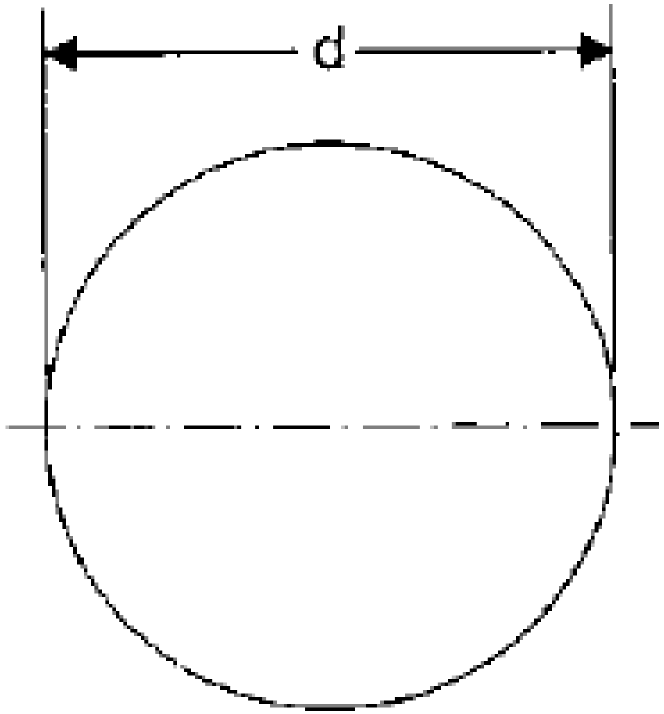
Section Modulus (Z)

3. Circular Section

$$I = \frac{\pi}{64} d^4$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{\pi}{32} d^3$$



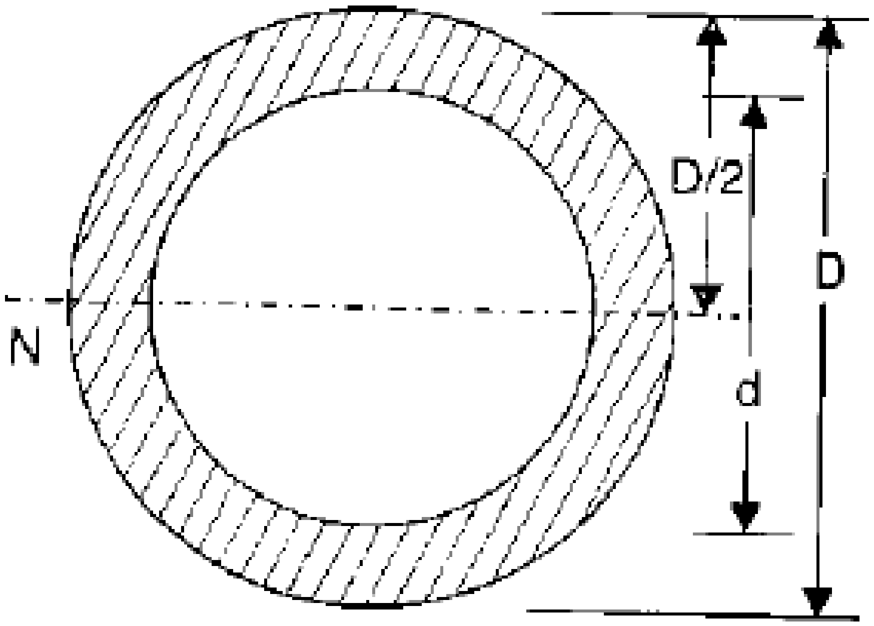
Section Modulus (Z)

4. Circular Hollow Section

$$I = \frac{\pi}{64}\,[D^4 - d^4]$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{\pi}{32D}\,[D^4 - d^4]$$

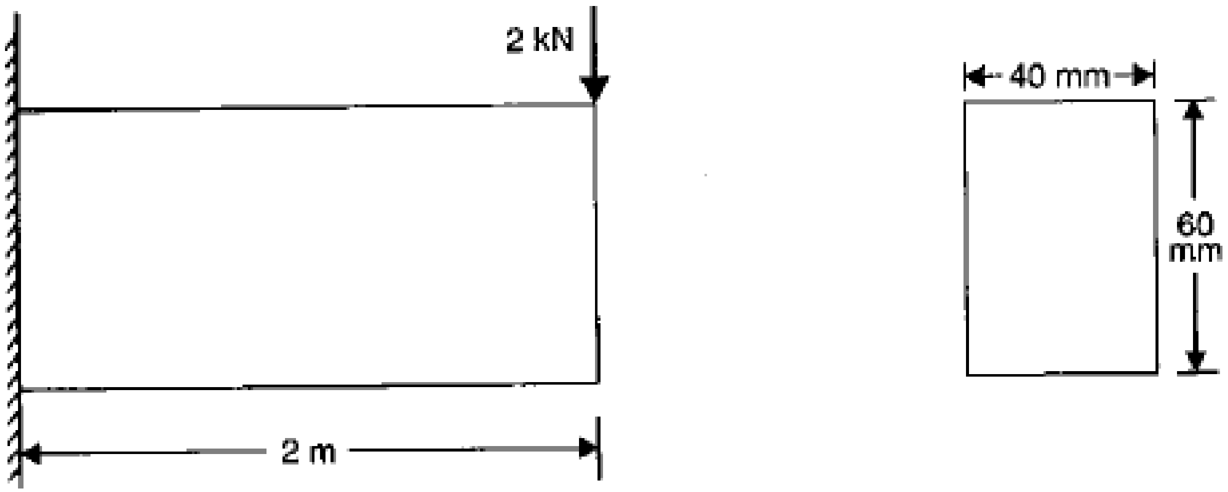


Problems

A cantilever of length 2m fails when a load of 2 kN is applied at the free end. If the section of the beam is 40 mm X 60 mm, find the stress at the failure.

Solution:

Problem Sketch:



- Let, σ_{max} = Stress at failure (Maximum stress)
- Since R is not given, we cannot use $\sigma = \frac{E}{R}y$
- Instead, use $M = \sigma_{max} \times I/y_{max}$ OR
- $M = \sigma_{max} \times Z$, where

$$Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6} = 24000 \text{ mm}^3$$

and, $M = W \times L = 2000 \times 2 \times 10^3 = 4 \times 10^6$ Nmm

Hence,

$$\sigma_{max} = \frac{M}{Z} = \frac{4 \times 10^6}{24000} = 166.67 \text{ N/mm}^2.$$

Problem

A square beam 20mm X 20mm in section and 2m long is simply supported at the ends. The beam fails when a point load of 400N is applied at the centre of the beam. What uniformly distributed load per meter length will break a cantilever of the same material 40mm wide, 60mm deep and 3m long?

Solution:

Square c/s.: 20 mm x 20 mm; L = 2m; W = 400 N

Rectangular c/s.: 40 mm x 60 mm; L = 3m; w = ?

Equate the maximum stress in the two cases.

Maximum stress in beam of square c/s.:

$$M = \sigma_{max} \times Z, \text{ where}$$

$$Z = \frac{bd^2}{6} = \frac{20 \times 20^2}{6} = \frac{4000}{3} \text{ mm}^3$$

$$M = \frac{w \times L}{4} = \frac{400 \times 2}{4} = 200 \text{ Nm}$$

$$= 200 \times 1000 = 200000 \text{ Nmm}$$

$$\sigma_{max} = \frac{200000 \times 3}{4000} = 150 \text{ N/mm}^2$$

Maximum stress in beam of rectangular c/s.:

$$M = \sigma_{max} \times Z, \text{ where}$$

Maximum bending moment for a cantilever beam loaded with UDL for the entire span is given by

$$M_{max} = wL^2 / 2$$

Hence, $M_{max} = \dfrac{w \times 3^2}{2} = 4.5w \text{ Nm}$

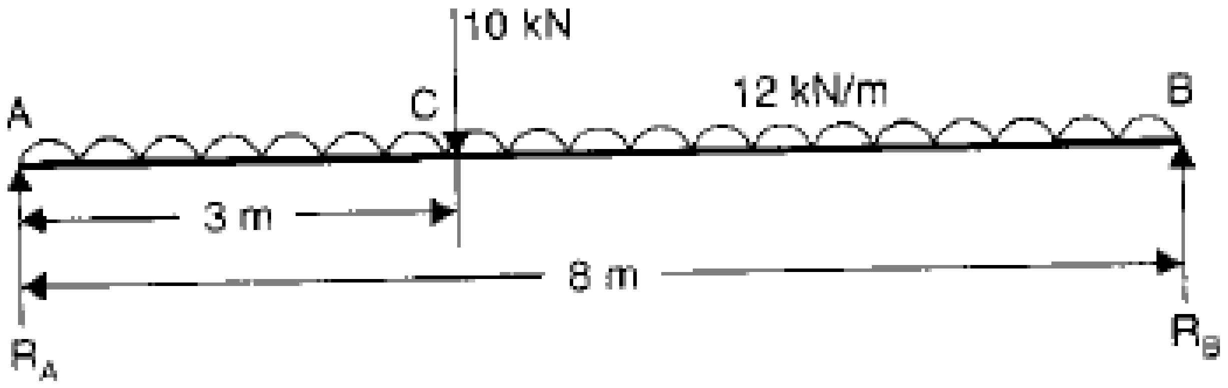
$$M_{max} = 4500 \text{ w N-mm}$$

$$
\begin{aligned}
M &= \sigma_{max} . Z \\
4.5 \times 1000w &= 150 \times 24000 \\
w &= \frac{150 \times 24000}{4.5 \times 1000} = \mathbf{800\ N/m.}
\end{aligned}
$$

Problem

A timber beam of rectangular section of length 8m is simply supported. The beam carries a U.D.L. of 12 kN/m run over the entire length and a point load of 10 kN at 3m from the left support. If the depth is two times the width and the stress in the timber is not to exceed 8 N/mm^2, find the suitable dimensions of the section.

Solution: Problem sketch



Given Data:

Length,	$L = 8$ m
U.D.L.,	$w = 12$ kN/m $= 12000$ N/m
Point load,	$W = 10$ kN $= 10000$ N
Depth of beam	$= 2 \times$ Width of beam
∴	$d = 2b$
Stress,	$\sigma_{max} = 8$ N/mm^2

Find the maximum bending moment in the beam and use $M = \sigma_{max} \times Z$ to find ***b*** and ***d***.

- Find the reaction forces at the supports
- Find the shear force at all the points of interest
- Find the maximum bending moment where SF is zero.

$$R_B \times 8 = 12000 \times 8 \times 4 + 10000 \times 3$$

$$R_B = \frac{12000 \times 32 + 30000}{8} = 51750 \text{ N}$$

$$R_A = \text{Total load} - R_B$$

$$= (12000 \times 8 + 10000) - 51750 = 54250 \text{ N}$$

Shear Forces:

F_B = - R_B = - 51750 N; F_C = +18250 N; F_A = +54250 N

Shear force changes its sign between B and C.

Let D be the point where SF = 0. Let ***x*** be the distance (in meters) of this point on the beam from B.

By calculation, x = 4.3125 m

- Find the BM at D.
- Find the Moment of Inertia (I) or section modulus (Z) of the beam's cross section
- Use the equation, M = σ_{max} x Z

$$M = R_B \times 4.3125 - 12000 \times 4.3125 \times \frac{4.3125}{2}$$

$$51750 \times 4.3125 - 111585.9375$$

$$111585.9375 \text{ Nm} = 111585.9375 \times 1000 \text{ Nmm}$$

$$Z = \frac{bd^2}{6} = \frac{b \times (2b)^2}{6} = \frac{2b^3}{3}$$

$$M = \sigma_{max} . Z$$

$$111585.9375 \times 1000 = 8 \times \frac{2b^3}{3}$$

Hence,
b = 275.5 mm
d = 551 mm

Any Questions ?

Thank you

WELLCOME TO MY PRESENTATION

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Subject : Strength of Materials

Sub:Code:67064

Chapter -01

- Topics: Understand simple stresses and strains

After the end of this lesson students will able to;

- State the theory of simple bending.
- Explain bending stresses.
- Identify position of neutral axis.
- Define moment of resistance.
- Define section modulus.
- Solve problems related to bending stresses in beam.

STRESS :

Stresses are expressed as the ratio of the applied force divided by the resisting area

Mathematically:

σ = Force / Area

Units:

N/m^2 or Pascal.

1kPa = 1000Pa, 1 Mpa= 10^6 Pa

TERMINOLOGIES RELATED TO STRESS

- **Stressor:**

A stressor is anything that has the effect of causing stress.

- **Stress capacity:**

While it is unclear precisely how much stress a person can carry, since each person has some stress in their lives, we say he/she has a capacity for stress. Similarly in case of Rocks, how much capacity they have to bear stress.

- **Stress-load:**

Everyone, even children, must carry some amount of stress in their daily lives. When we think of stress as having an amount, or quantity, we refer to this as the person's stress-load. And here in case of rocks, we say that how much an already existing stress is applied on a rock.

TYPES OF STRESS

There are two types of stress

1) Normal Stress

1.1) Tensile stress

2.2) Compressive stress

2) Combine Stress

2.1) Shear stress

2.2)Tortional stress

1)Normal Stress:

The resisting area is perpendicular to the applied force

1) Tensile Stress:

- **It is a stress induced in a body when it is subjected to two equal and opposite pulls (Tensile force) as a result of which there is tendency in increase in length.**
- **It acts normal to the area and pulls on the area.**

2) Compressive Stress:

- **Stress induced in a body, when subjected to two equal and opposite pushes as a result of which there is a tendency of decrease in length of the body.**
- **It acts normal to the area and it pushes on the area.**

2) Combined Stress:

A condition of stress that cannot be represented by a single resultant stress.

2.1) Shear stress:

- **Forces parallel to the area resisting the force cause shearing stress.**
- **It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act.**
- **Shearing stress is also known as tangential stress**

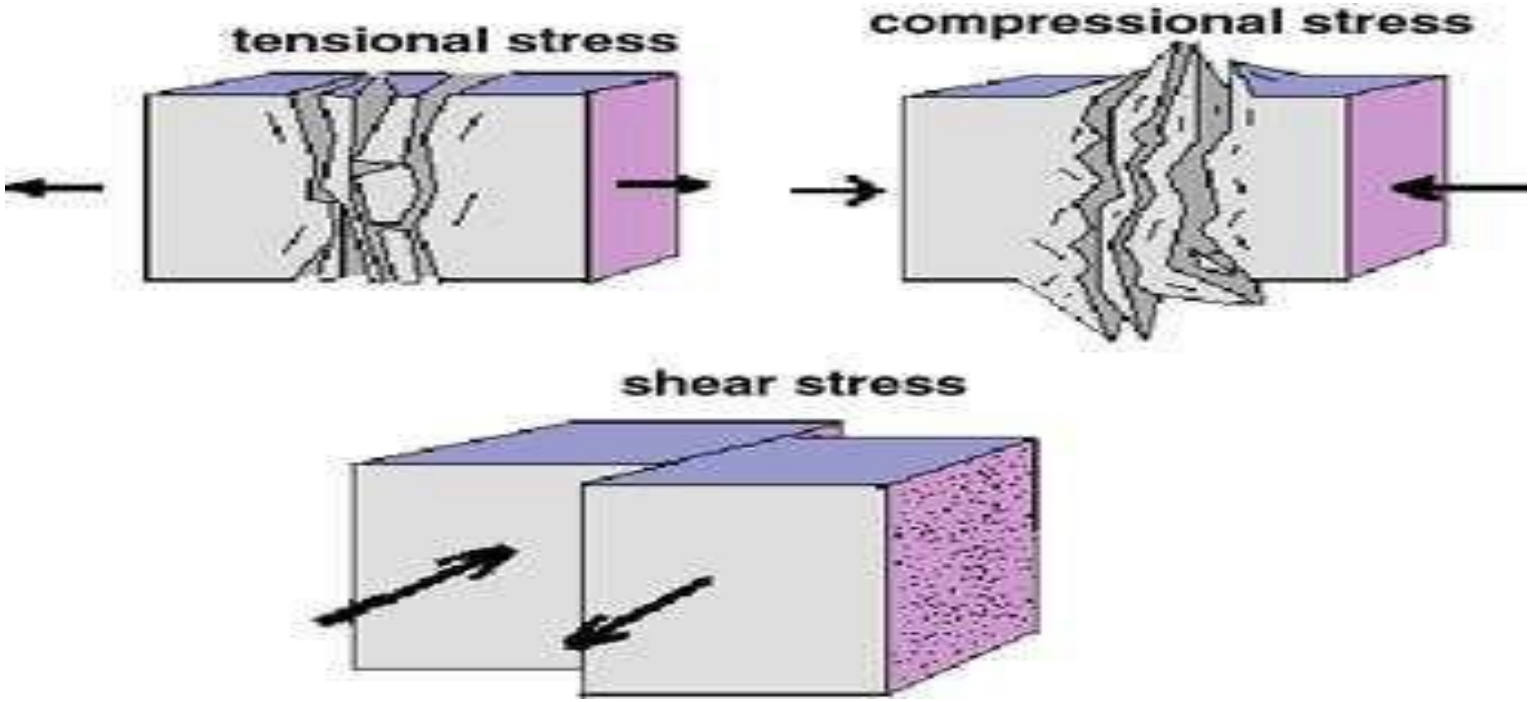
.

2.2) Tortional stress:

The stresses and deformations induced in a circular shaft by a twisting moment.

TYPES OF STRESS:

Diagrams



STRAIN:

When a body is subjected to some external force, there is some change in the dimension of the body. The ratio of change in dimension of body to its original dimension is called as strain.

Strain is a dimensionless quantity.

TERMINOLOGIES RELATED TO STRAIN:

- **Longitudinal or Linear Strain**

  Strain that changes the length of a line without changing its direction. Can be either compressional or tensional.

- **Compression**

  Longitudinal strain that shortens an object.

- **Tension**

  Longitudinal strain that lengthens an object.

➢Shear

❖ Strain that changes the angles of an object.

❖Shear causes lines to rotate.

➢Infinitesimal Strain

❖ Strain that is tiny, a few percent or less.

❖ Allows a number of useful mathematical simplifications and approximations.

➢Finite Strain

❖ Strain larger than a few percent.

❖Requires a more complicated mathematical treatment than infinitesimal strain.

➢Homogeneous Strain

- Uniform strain.
- Straight lines in the original object remain straight.
- Parallel lines remain parallel.
- Circles deform to ellipses.
- Note that this definition rules out folding, since an originally straight layer has to remain straight.

➢Inhomogeneous Strain

- How real geology behaves.
- Deformation varies from place to place.
- Lines may bend and do not necessarily remain parallel.

TYPES OF STRAIN

1. Tensile Strain
2. Compression Strain
3. Volumetric Strain
4. Shear Strain

1) Tensile Strain:

Ratio of increase in length to the original length of the body when it is subjected to a pull force.

Tensile strain = Increase in length/ Original Length

= dL/L

2) Compressive Strain:

Ratio of decrease in Length to the original length of body when it is subjected to push force.

Compressional Strain = Decrease in length/Original Length

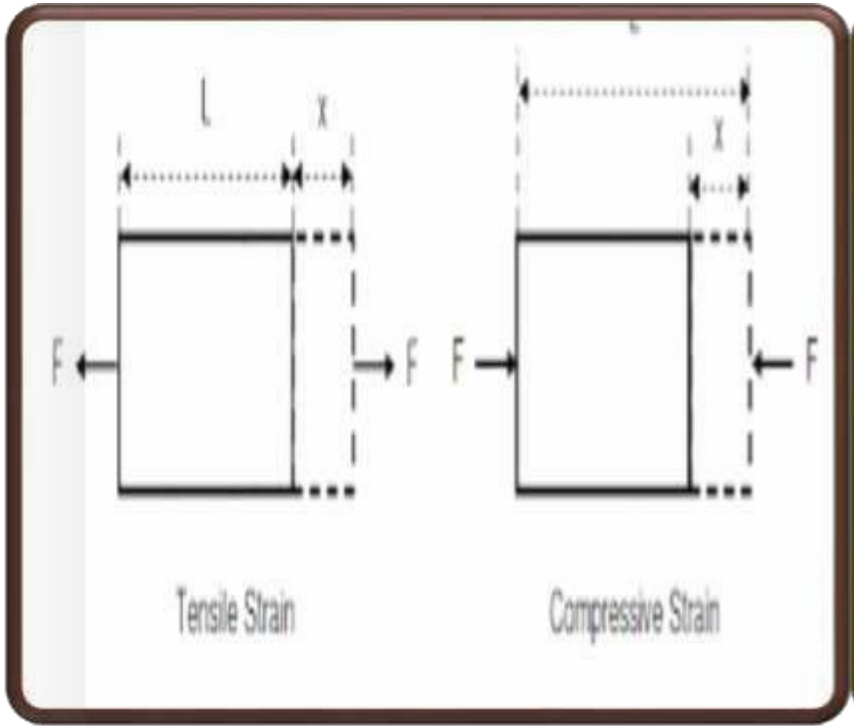
= dL/L

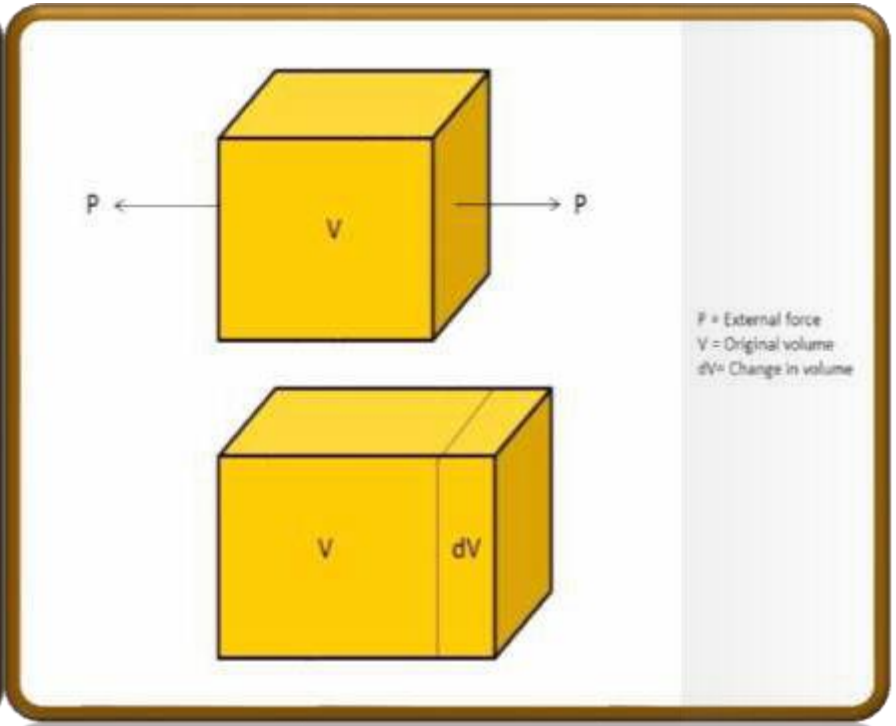
3) Volumetric Strain:

Ratio of change of volume to the original volume.

Volumetric Strain= dV/V

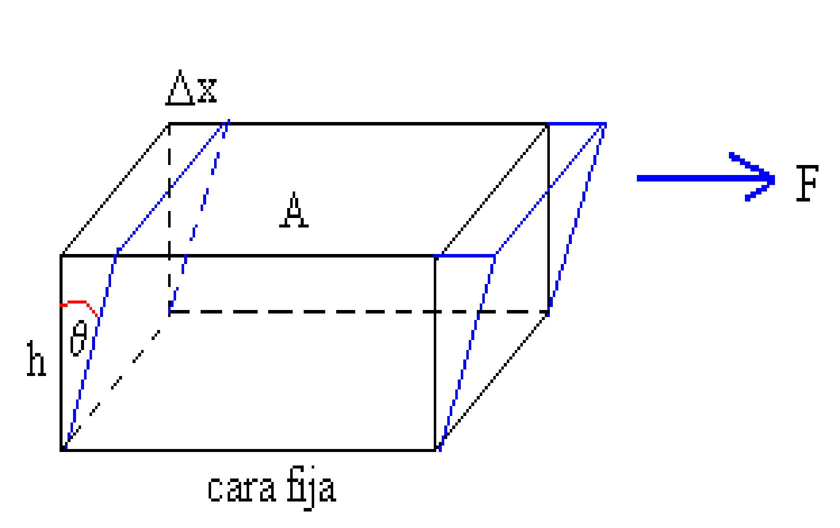
Types of Strain Diagrams





4) Shear Strain

Strain due to shear stresses.



Sign convection for direct strain

- ❖Tensile strains are considered positive in case of producing increase
- ❖ in length.
- ❖Compressive strains are considered negative in case of producing
- ❖decrease in length.

Relation between Stress and Strain

Hooke's Law:

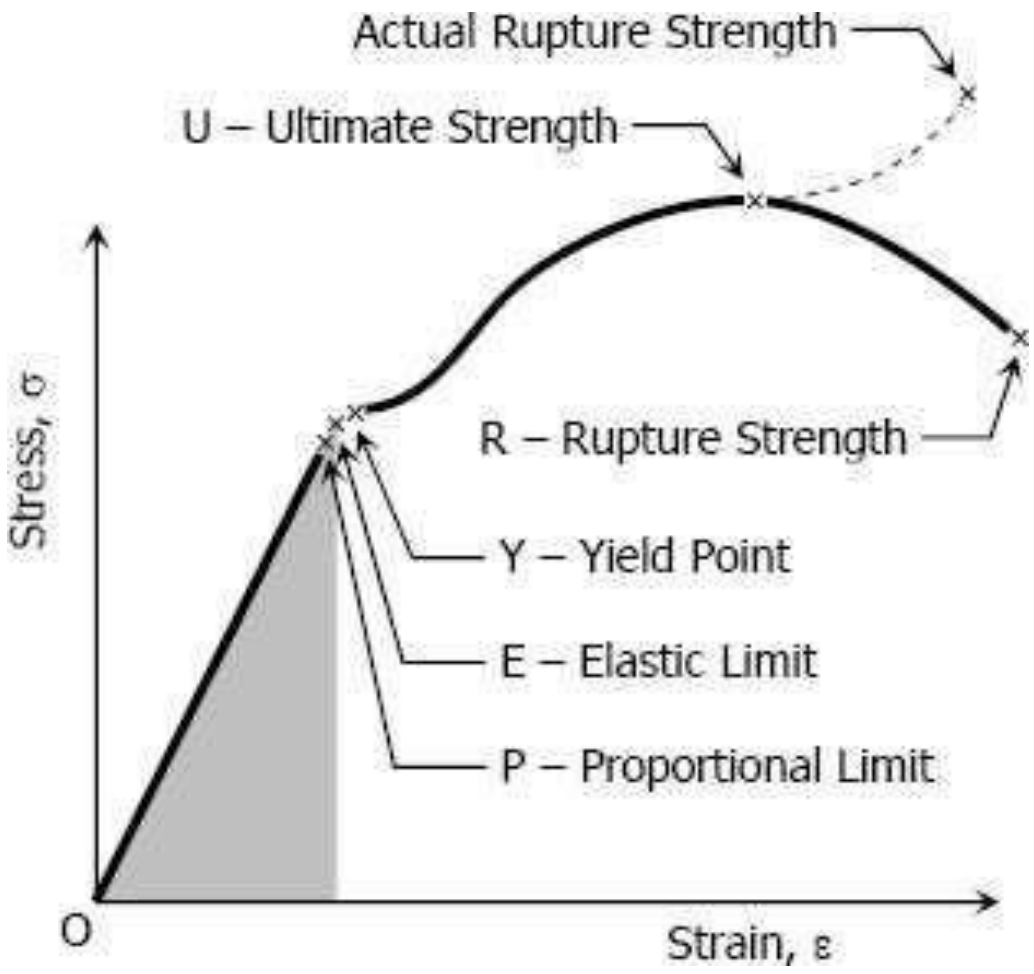
"Within Elastic limit the ratio of stress applied to strain developed is constant".
The constant is known as Modulus of elasticity or Young's Modulus or Elastic Modulus.

Mathematically:

E= Stress/Strain

- **Young's Modulus E, is generally assumed to be same in tension or Compression and for most of engineering application has high Numerical value.**
- **Typically E=210×10*9 N/m*2 for steel**

STRESS AND STRAIN DIAGRAM



Any Questions ?

Thank you

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Chapter -03

- Topics: Understand the thermal stresses and strains

After the end of this lesson students will able to;

- Define thermal stresses and strains.
- Explain the method of thermal stresses in simple bars, circular tapering section and bars of varying section.
- Explain thermal stresses in composite bar.
- Superposition of thermal stresses.
- Solve problems related to thermal stresses.

Thermal effects

- Changes in temperature produce expansion or contraction of materials and result in *thermal strains* and *thermal stresses*
- For most structural materials, thermal strain ε_T is proportional to the temperature change ΔT :

$$\varepsilon_T = \alpha (\Delta T)$$

coefficient of thermal expansion

FIG. 2-19 Block of material subjected to an increase in temperature

- When a sign convention is needed for thermal strains, we usually assume that expansion is positive and contraction is negative

Thermal Stress

- Suppose we have a bar subjected to an axial load. We will then have:

$$\varepsilon = \sigma / E$$

- Also suppose that we have an identical bar subjected to a temperature change ΔT.

We will then have:

$$\varepsilon_T = \alpha (\Delta T)$$

- Equating the above two strains we will get:

$$\sigma = E \alpha (\Delta T)$$

- We now have a relation between axial stress and change in temperature

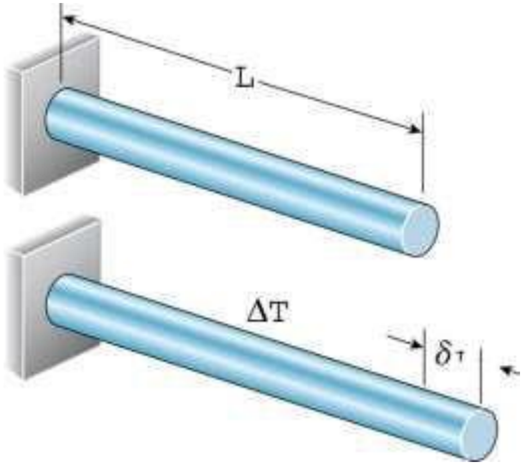


FIG. 2-20 Increase in length of a prismatic bar due to a uniform increase in temperature (Eq. 2-16)

• Assume that the material is homogeneous and isotropic and that the temperature increase ΔT is uniform throughout the block.

• We can calculate the increase in *any* dimension of the block by multiplying the original dimension by the thermal strain.

$$\delta_T = \varepsilon_T L = \alpha (\Delta T) L$$

Temperature – Displacement relation

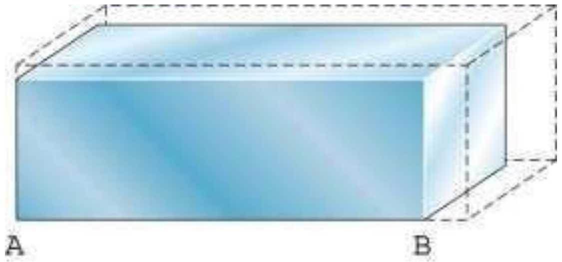


FIG. 2-19 Block of material subjected to an increase in temperature

Thermal Strain

As in the case of lateral strains, thermal strains do not induce stresses unless they are constrained.

The total strain in a body experiencing thermal stress may be divided into two components:

Strain due to stress, ε_σ and

That due to temperature, ε_T .

Thus: $\varepsilon = \varepsilon_\sigma + \varepsilon_T$

$$\varepsilon = \frac{\sigma}{E} + \alpha T$$

❖Thermal Stresses In Bars of Tapering Section:-

➢ Taken circular bar uniformly tapering which has:

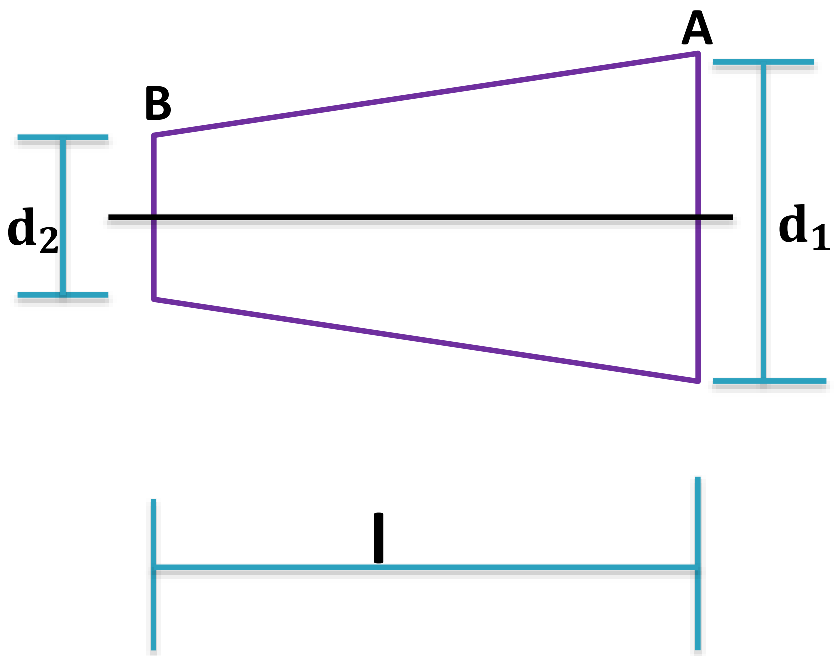
l = length of bar

d_1 = Dia of bigger end of bar

d_2 = Dia of smaller end of bar

t = change in temperature

α = coefficient of thermal expansion.



➢ Derive the equation of thermal stress for tapering section ;

- Due to contraction Compressive force P acted when temperature will be rise ;

$$\delta l = \frac{4Pl}{\pi E d_1 d_2} \quad \text{(1)}$$

- Due to expansion when temperature will be rise that time;

$$\delta l = l.\alpha.t \quad \text{(2)}$$

- Here no change in length in equation (1) & (2),

so, $$\frac{4Pl}{\pi E d_1 d_2} = l.\alpha.t \rightarrow \mathbf{P} = \frac{(l.\alpha.t) * \pi E d_1 d_2}{4l}$$

Max. stress = $$\sigma = \frac{P}{\frac{\pi d2^2}{4}} \rightarrow \sigma = \alpha.t.E\frac{d_1}{d_2}$$

Thermal Stress in Bars of Varying section

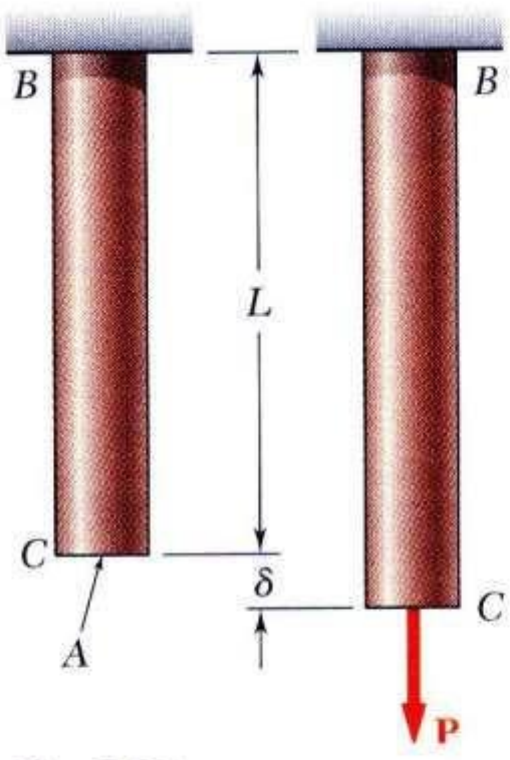


Fig. 2.22

- From Hooke's Law:

$$\sigma = E\varepsilon \qquad \varepsilon = \frac{\sigma}{E} = \frac{P}{AE}$$

- From the definition of strain:

$$\varepsilon = \frac{\delta}{L}$$

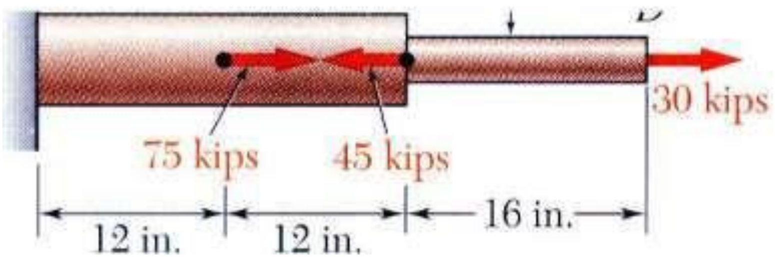
- Equating and solving for the deformation,

$$\delta = \frac{PL}{AE}$$

- With variations in loading, cross-section or material properties,

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Example



$E = 29 \times 10^{-6}$ psi

$D = 1.07$ in. $d = 0.618$ in.

Determine the deformation of the steel rod shown under the given loads.

- SOLUTION:
  - Divide the rod into components at the load application points.
  - Apply a free-body analysis on each component to determine the internal force
  - Evaluate the total of the component deflections.

SOLUTION:

- Divide the rod into three components:

$$L_1 = L_2 = 12 \text{ in.} \qquad L_3 = 16 \text{ in.}$$

$$A_1 = A_2 = 0.9 \text{ in}^2 \qquad A_3 = 0.3 \text{ in}^2$$

- Apply free-body analysis to each component to determine internal forces,

$$P_1 = 60 \times 10^3 \text{ lb}$$

$$P_2 = -15 \times 10^3 \text{ lb}$$

$$P_3 = 30 \times 10^3 \text{ lb}$$

- Evaluate total deflection,

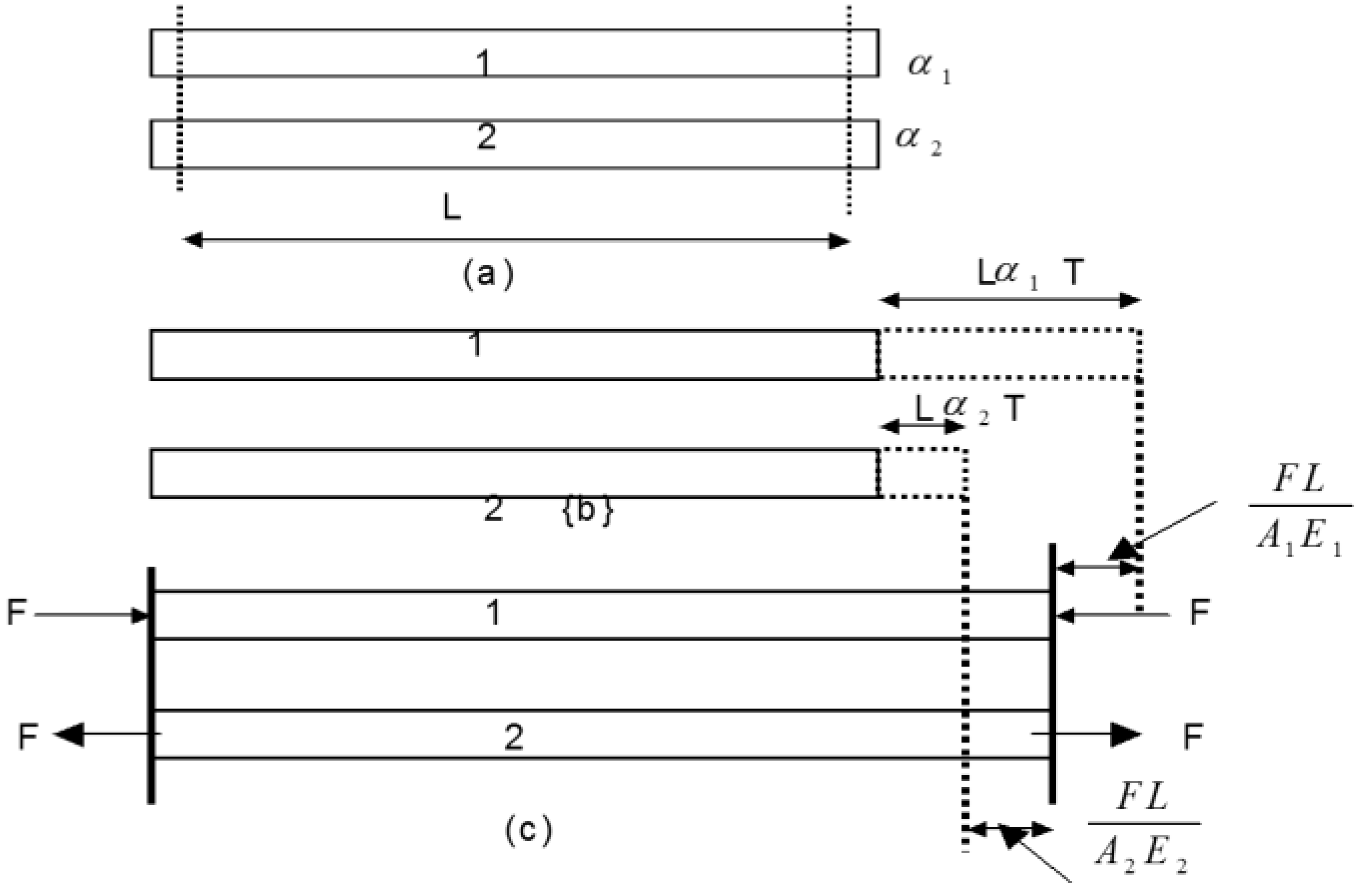
$$\delta = \sum \frac{PL}{AE} = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$

$$= \frac{1}{29 \times 10^6}\left[\frac{\left(60 \times 10^3\right)12}{0.9} + \frac{\left(-15 \times 10^3\right)12}{0.9} + \frac{\left(30 \times 10^3\right)16}{0.3}\right]$$

$$= 75.9 \times 10^{-3} \text{ in.}$$

$$\boxed{\delta = 75.9 \times 10^{-3} \text{ in.}}$$

Temperature stresses in composite bars

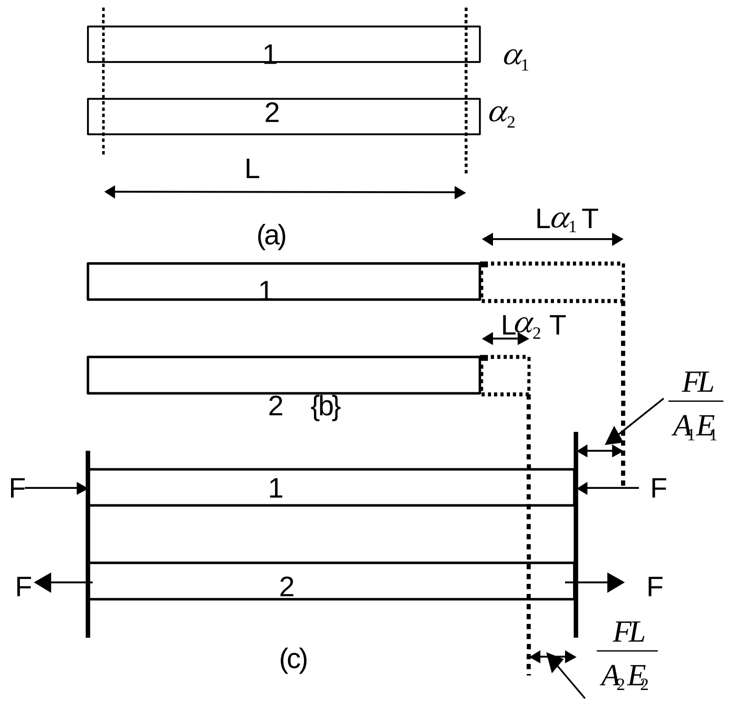


Temperature Stresses Contd.

Free expansions in bars (1) and (2) are $L\alpha_1 T$ *and* $L\alpha_2 T$ respectively.

Due to end fixing force, F: the decrease in length of bar (1) is

$\frac{FL}{A_1E_1}$ and the increase in length of (2) is $\frac{FL}{A_2E_2}$.



At Equilibrium:

$$L\alpha_1 T - \frac{FL}{A_1E_1} = L\alpha_2 T + \frac{FL}{A_2E_2}$$

$$i.e. \quad F\left[\frac{1}{A_1E_1} + \frac{1}{A_2E_2}\right] = T(\alpha_1 - \alpha_2)$$

$$\sigma_1 = \frac{T(\alpha_1 - \alpha_2)A_2E_1E_2}{A_1E_1 + A_2E_2}$$

$$\sigma_2 = \frac{T(\alpha_1 - \alpha_2)A_1E_1E_2}{A_1E_1 + A_2E_2}$$

Note: As a result of Force, F, bar (1) will be in compression while (2) will be in tension.

Any Questions ?

Thank you

WELLCOME TO MY PRESENTATION

<u>Presented By</u>

Ariful Islam

Work Shop Superintendent

(Tech/Mechanical)

Mechanical Technology

Mymensingh Polytechnic Institute

Subject : Strength of Materials

Sub:Code:67064

Chapter -05

Topics: Understand the analysis of the effects of loading on beam.

After the end of this lesson students will able to;

1. Define beams and classify it.
2. Distinguish between statically determinate and statically indeterminate beams.
3. Define bending moment and shear force.
4. Identify positive sign and negative sign of bending moment and shear force.
5. Express the relation between bending moment and shear force.
6. Define deformed sections, inflection point and locate their positions.
7. Draw shear force diagram and bending moment diagram of beams.
8. Solve problems related to beam. the theory of simple bending.

Why study stresses in beams

What are beams

- A structural member which is long when compared with its lateral dimensions, subjected to transverse forces so applied as to induce bending of the member in an axial plane, is called a beam.

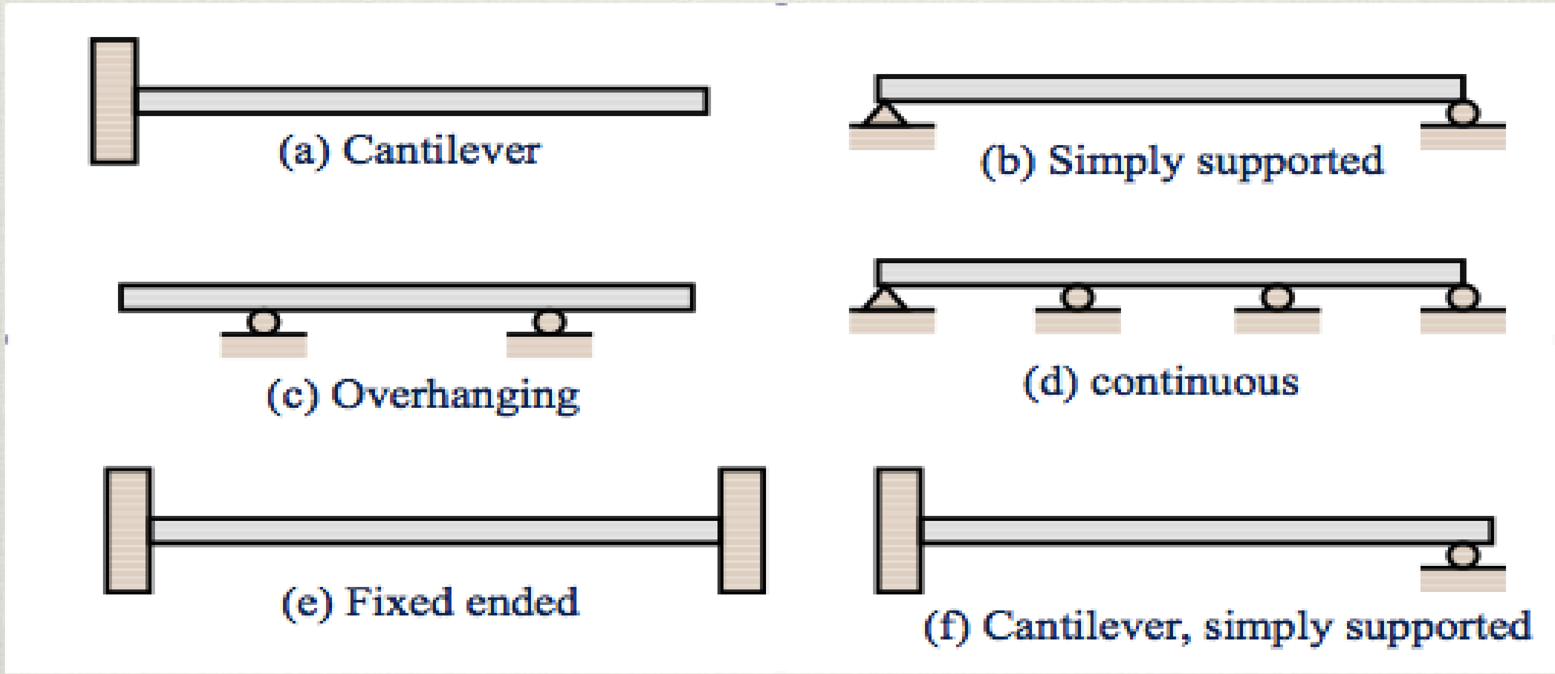
Objective

- When a beam is loaded by forces or couples, stresses and strains are created throughout the interior of the beam.

- To determine these stresses and strains, the internal forces and internal couples that act on the cross sections of the beam must be found.

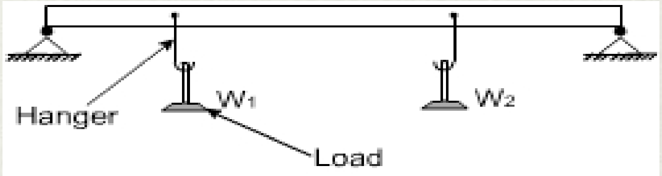
Beam Types

- Types of beams- depending on how they are supported.

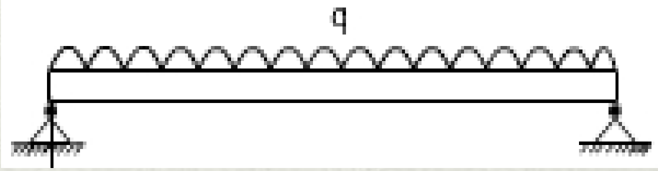


Load Types on Beams

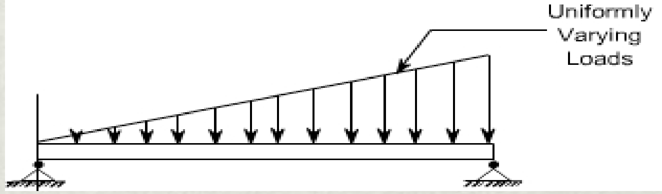
- Types of loads on beam
  - Concentrated or point load



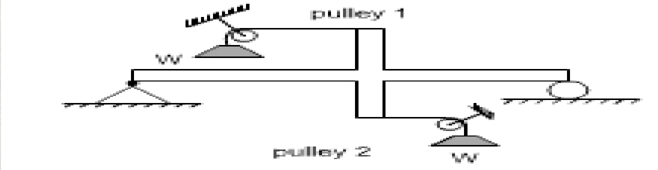
  - Uniformly distributed load



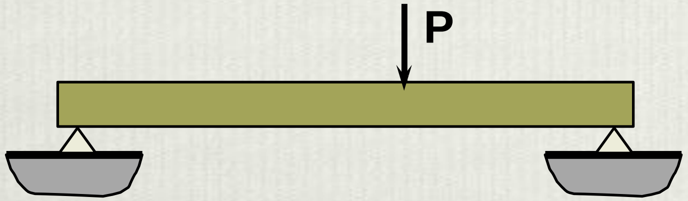
  - Uniformly varying load

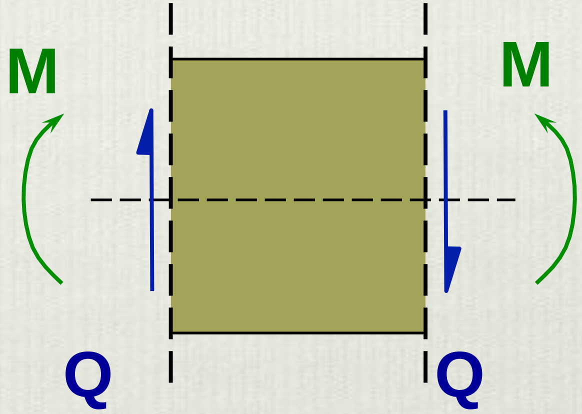


  - Concentrated Moment

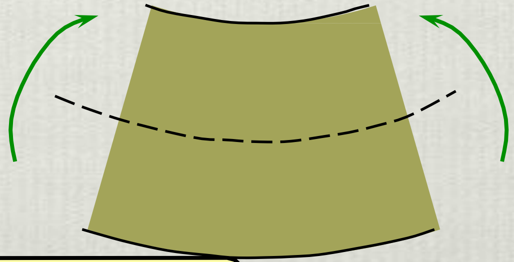


Sign Convention for forces and moments



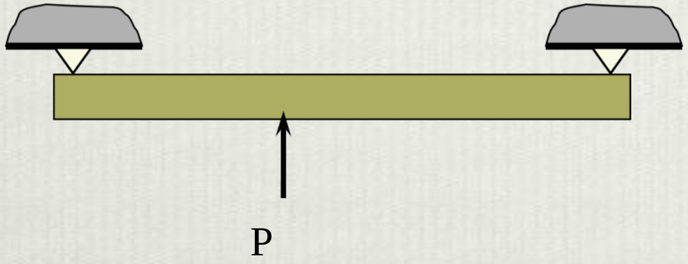


"Happy" Beam is +VE

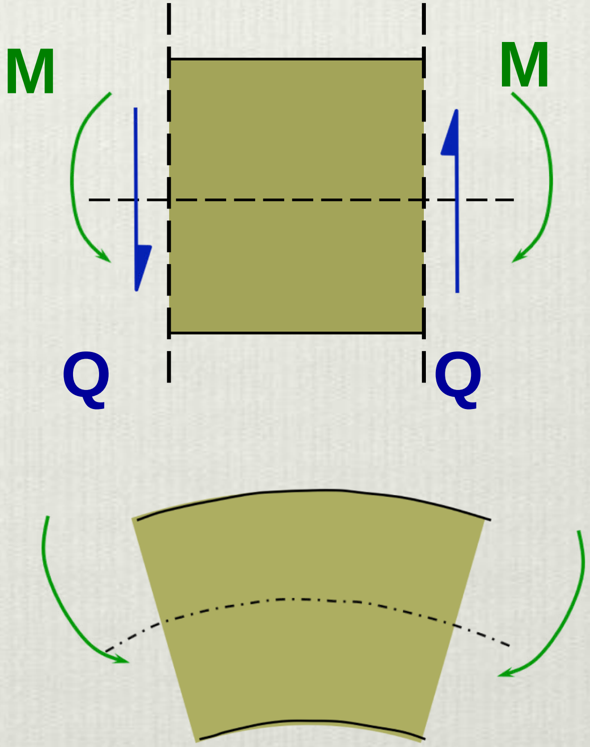


+VE (POSITIVE)

Sign Convention for forces and moments



"Sad" Beam is -VE



-VE (POSITIVE)

Sign Convention for forces and moments

- Positive directions are denoted by an internal shear force that causes clockwise rotation of the member on which it acts, and an internal moment that causes compression, or pushing on the upper arm of the member.

- Loads that are opposite to these are considered negative.

SHEAR FORCES AND BENDING MOMENTS

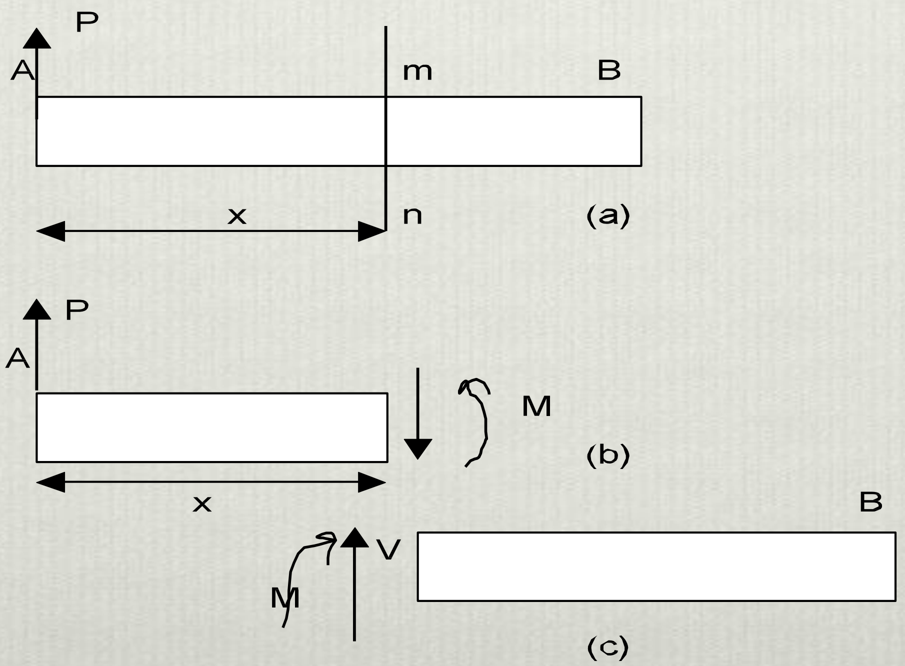
- The resultant of the stresses must be such as to maintain the equilibrium of the free body.
- The resultant of the stresses acting on the cross section can be reduced to a shear force and a bending moment.
- The stress resultants in statically determinate beams can be calculated from equations of equilibrium.

Shear Force and Bending Moment in a Beam

Summing forces in the vertical direction and also taking moments about the cut section:

$$\sum Fx = 0 \text{ i.e. } P - V = 0 \text{ or } V = P$$

$$\sum M = 0 \text{ i.e. } M - Px \text{ or } M = Px$$



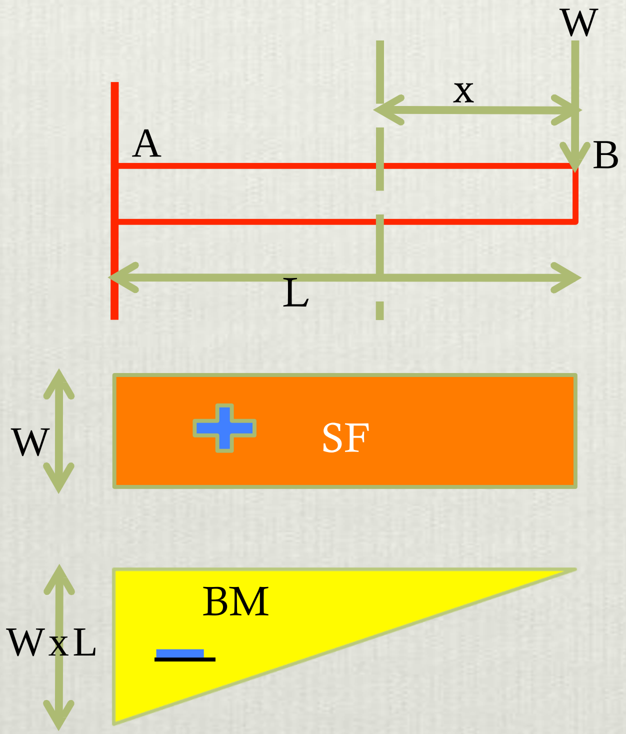
Shear Force and Bending Moment

- **Shear Force:** is the algebraic sum of the vertical forces acting to the left or right of the cut section

- **Bending Moment:** is the algebraic sum of the moment of the forces to the left or to the right of the section taken about the section

SF and BM formulas

Cantilever with point load



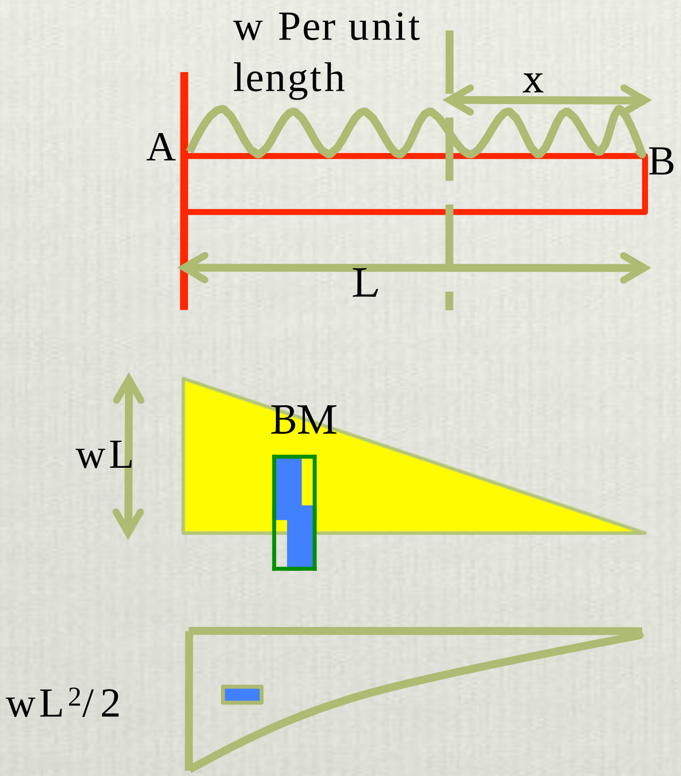
F_x= Shear force at X
M_x= Bending Moment at X

$F_x=+W$

$M_x=-Wx$
at $x=0 \Rightarrow M_x=0$
at $x=L \Rightarrow M_x=-WL$

SF and BM formulas

Cantilever with uniform distributed load



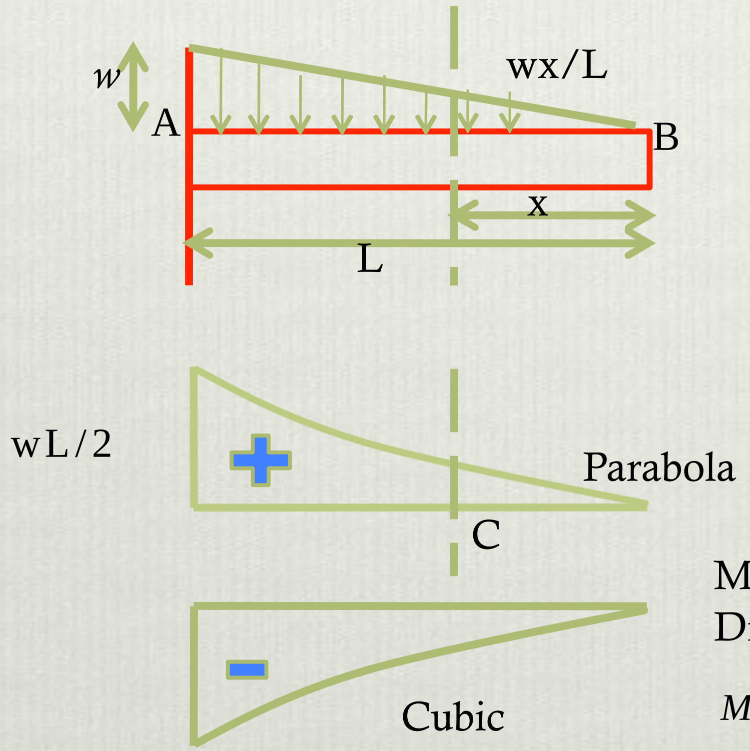
F_x= Shear force at X
M_x= Bending Moment at X

$F_x=+wx$
at x=0 $F_x=0$
at x=L $F_x=wL$

M_x=-(total load on right portion)*
Distance of C.G of right portion
$M_x=-(wx).x/2=-wx^2/2$
at x=0=> $M_x=0$
at x=L=> $M_x=-wl^2/2$

SF and BM formulas

Cantilever with gradually varying load



F_x= Shear force at X
M_x= Bending Moment at X

$$F_x = \frac{wx^2}{2L}$$

at x=0 F_x=0
at x=L F_x=$wL/2$

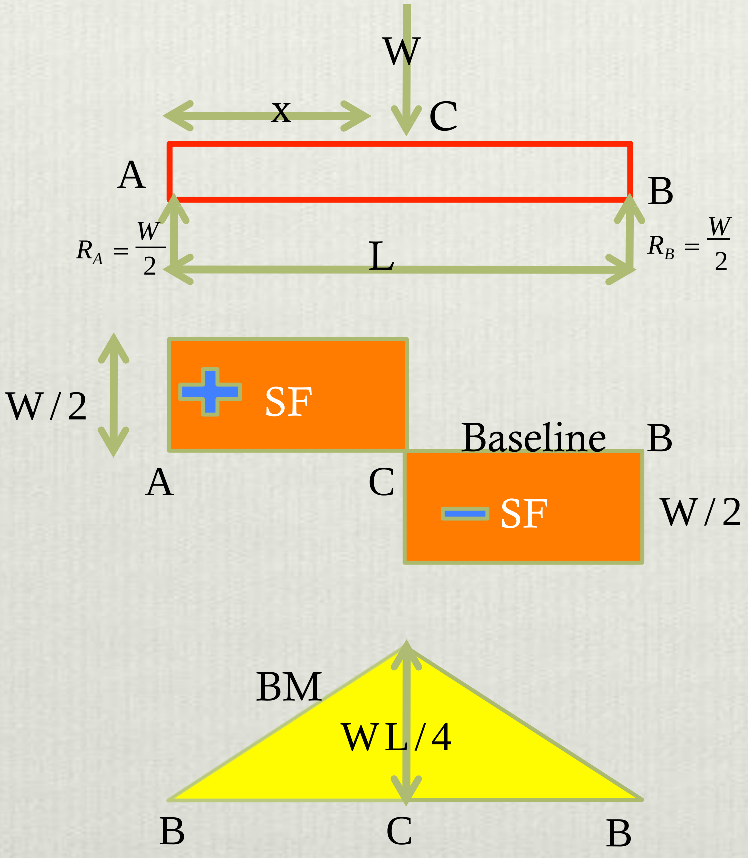
M_x=-(total load for length x)*
Distance of load from X

$$M_x = \frac{wx^3}{6L}$$

at x=0=> M_x=0
at x=L=> M_x=$-wl^2/6$

SF and BM formulas

Simply supported with point load



F_x= Shear force at X
M_x= Bending Moment at X

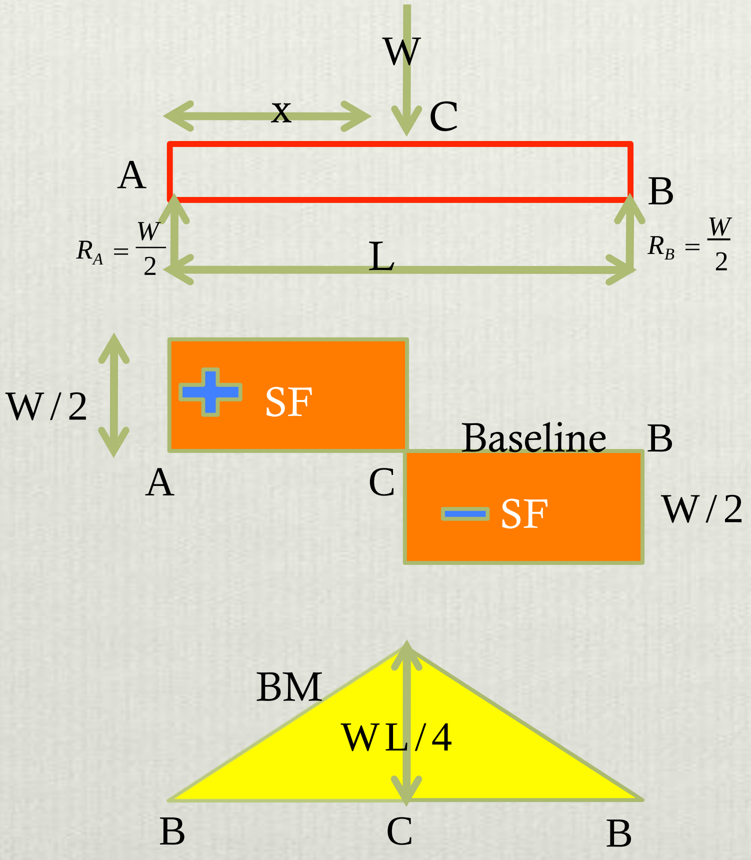
F_x=+W/2 (SF between A & C)
Resultant force on the left portion

$$\left(\frac{W}{2} - W\right) = -\frac{W}{2}$$

Constant force between B to C

SF and BM formulas

Simply supported with point load



F_x= Shear force at X
M_x= Bending Moment at X

for section
between A & C

$$M_x = R_A x = \frac{W}{2} x$$

at A x=0=> M_A=0
at C x=L/2=> $M_C = \frac{W}{2} \times \frac{L}{2}$

for section
between C & B

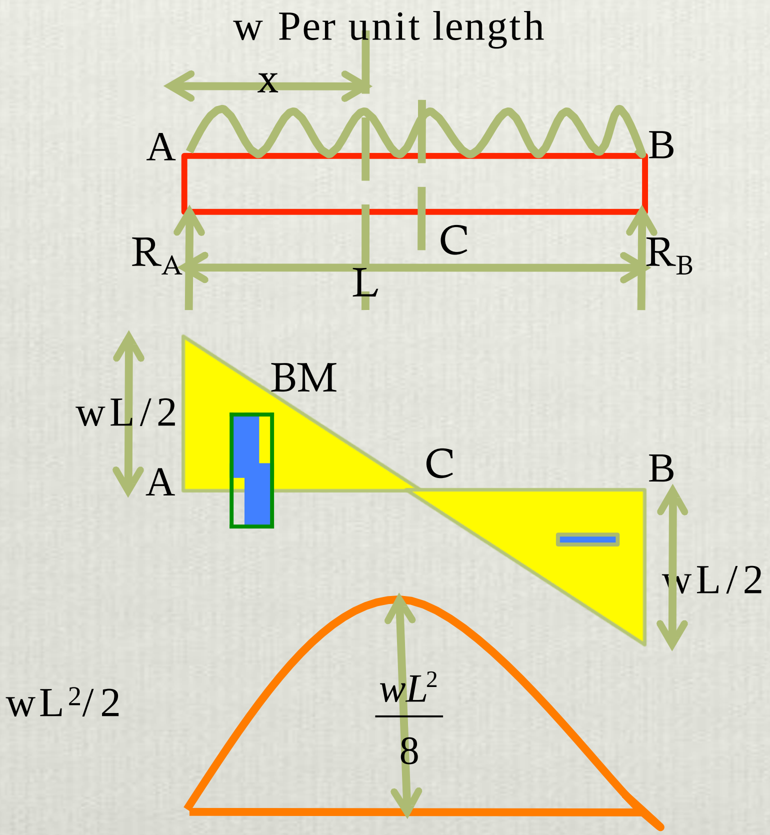
$$M_x = R_A x - W \times \left(x - \frac{L}{2} \right) = \frac{W}{2} x - Wx + W \frac{L}{2}$$

$$= -\frac{W}{2} x + W \frac{L}{2}$$

$$M_B = \frac{WL}{2} - \frac{W}{2} L = 0$$

SF and BM formulas

Simply supported with uniform distributed load



F_x= Shear force at X

M_x= Bending Moment at X

$$R_A = R_B = \frac{wL}{2}$$

$$F_x = R_A - w.x = \frac{wL}{2} - w.x$$

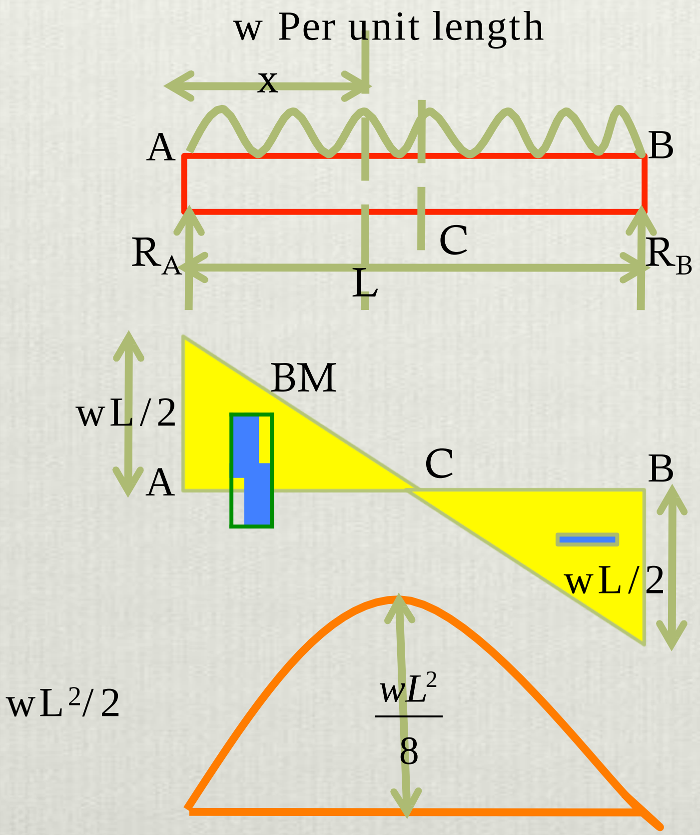
$$x = 0 \Rightarrow F_A = \frac{wL}{2} - \frac{w.0}{2} = \frac{wL}{2}$$

$$x = \frac{L}{2} \qquad F_C = \frac{wL}{2} - \frac{wL}{2} = 0$$

$$x = L \Rightarrow F_B = \frac{wL}{2} - wL = \frac{wL}{2}$$

SF and BM formulas

Simply supported with uniform distributed load



F_x= Shear force at X

M_x= Bending Moment at X

$$M_x = R_A x - w.x\frac{x}{2}$$

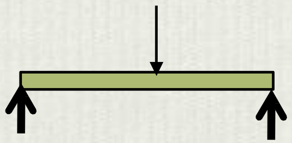
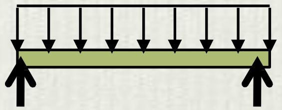
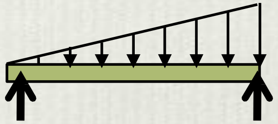
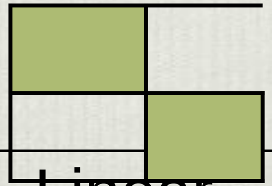
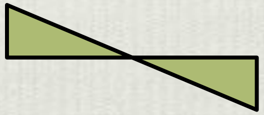
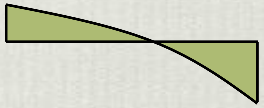
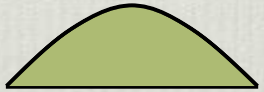
$$= \frac{wL}{2}x - \frac{w.x^2}{2}$$

$$x = 0 \Rightarrow M_A = \frac{wL}{2}.0 - \frac{w.0}{2} = 0$$

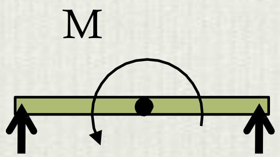
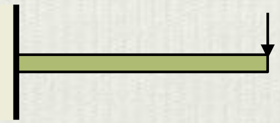
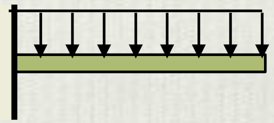
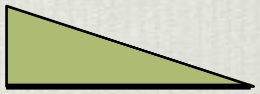
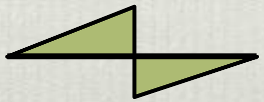
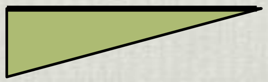
$$x = \frac{L}{2} \quad M_c = \frac{wL}{2}.\frac{L}{2} - \frac{w}{2}\left(\frac{L}{2}\right)^2 = \frac{wL^2}{4} - \frac{wL^2}{8} = \frac{wL^2}{8}$$

$$x = L \Rightarrow \quad M_B = \frac{wL}{2}L - \frac{w}{2}L^2 = 0$$

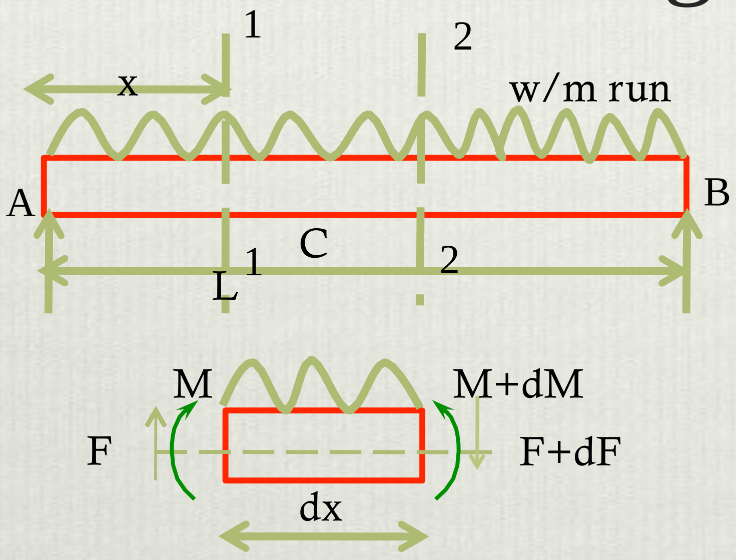
SF and BM diagram

Load	P	Constant	Linear
			
Shear	Constant	Linear	Parabolic
			
Moment	Linear	Parabolic	Cubic
			

SF and BM diagram

Load	0	0	Constant
			
Shear	Constant	Constant	Linear
			
Moment	Linear	Linear	Parabolic
			

Relation between load, shear force and bending moment



$$\frac{dF}{dx} = -w$$

The rate of change of shear force is equal to the rate of loading

$$\frac{dM}{dx} = F$$

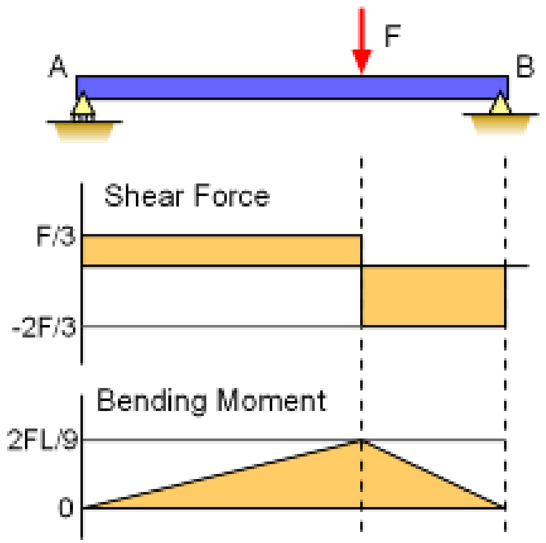
The rate of change of bending moment is equal to the shear force at the section

Procedure for determining shear force and bending moment diagrams

- Compute the support reactions from the free-body diagram (FBD) of the entire beam.
- Divide the beam into segment so that the loading within each segment is continuous.
- Draw a FBD for the part of the beam lying either to the left or to the right of the cutting plane, whichever is more convenient. At the cut section, show V and M acting in their positive directions.
- Determine the expressions for Shear force (V) and M from the equilibrium equations obtain from the FBD

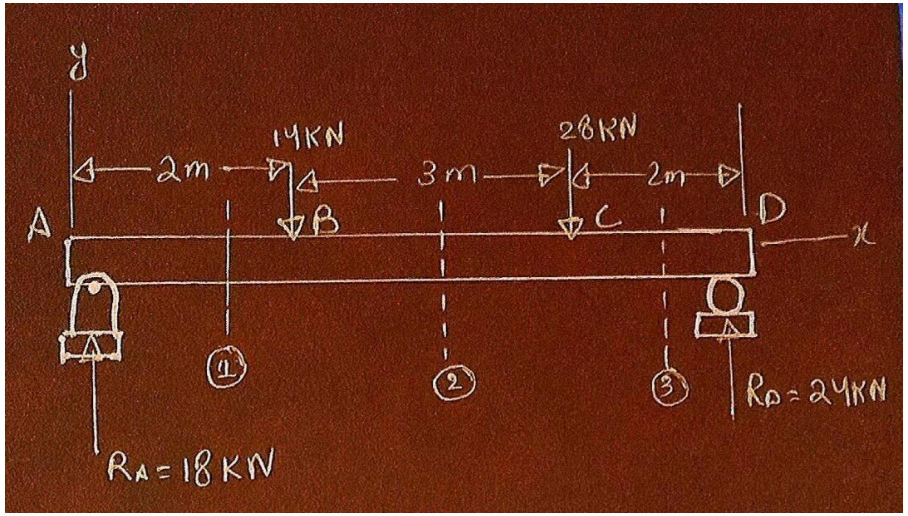
Cont…

- It is visually desirable to draw the shear force V-diagram below the FBD of the entire beam, and then draw the M- diagram below the V-diagram



Sample problem

The simply supported beam in Fig. (a) carries two concentrated loads. (1) Derive the expressions for the shear force and the bending moment for each segment of the beam. (2) Sketch the shear force and bending moment diagrams. Neglect the weight of the beam. Note that the support reactions at A and D have been computed and are shown in Fig. (a).



Solution:

Part 1

The determination of the expressions for V and M for each of the three beam segments (AB,BC, and CD) is explained below

Segment AB (0<x<2 m)

ΣFy =0 +↑

18-V = 0

<u>V = +18 kN</u>

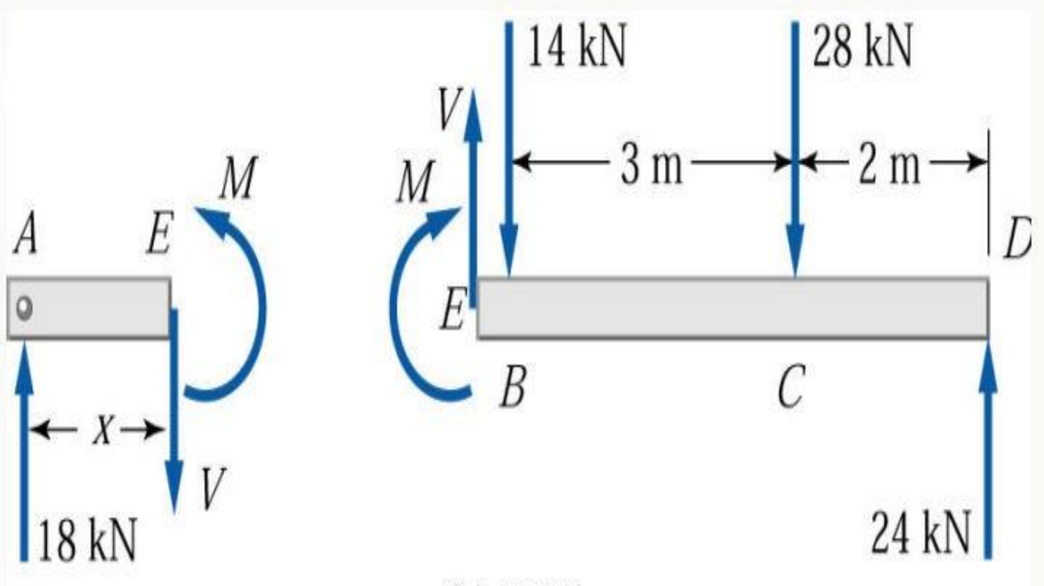
ΣM(at E) = 0 (CCW +)

- 18x+ M = 0

<u>M = +18x kN· m</u>

The point E is at

Position 1 in fig(a)



(b) FBDs

Segment BC ($2<x<5$)

$\Sigma F_y = 0$ +↑

18-14-V = 0

V = +18-14 =

+4 kN Answer

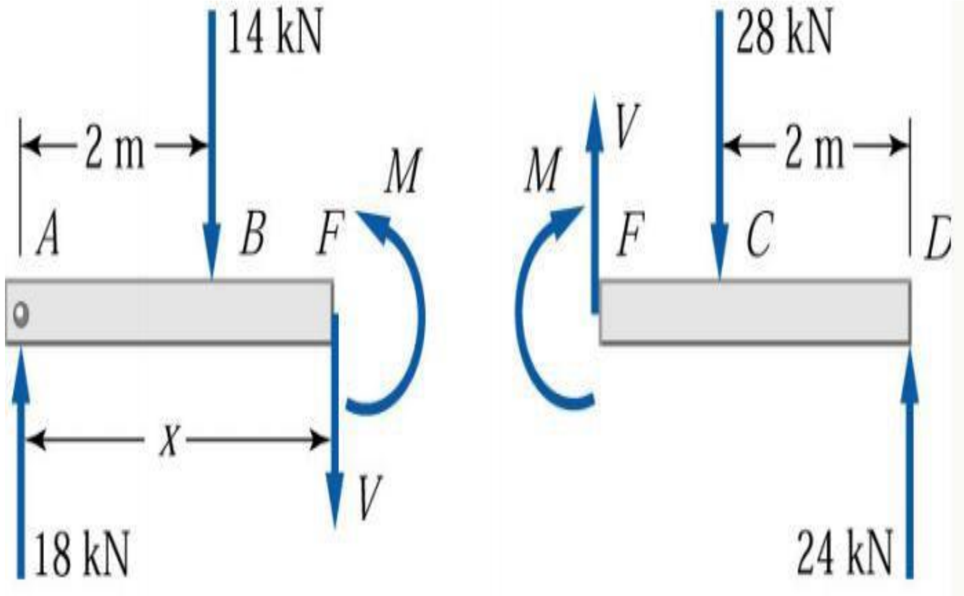
ΣM(at f) = 0 CCW +

- 18x + 14(x-2) + M = 0

M = +18x-14(x-2)

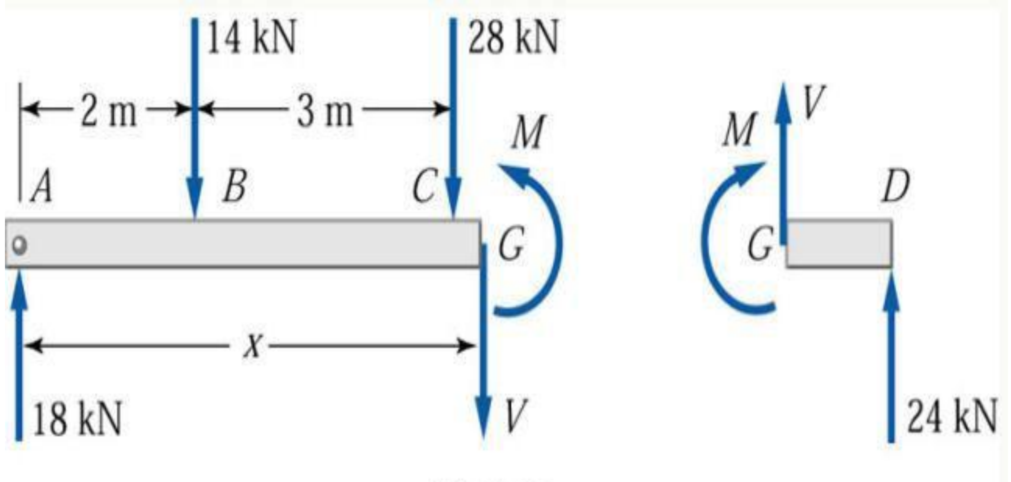
= 4x+28 kN· m

Point F is at position 2 in fig a



(c) FBDs

Segment CD (5 m < x < 7 m)



(d) FBDs

ΣFy =0 +↑

18-14—28-V = 0

V = +18-14-28

= <u>-24 kN Answer</u>

ΣM(at G) = 0 CCW +

-18x+ 14(x-2)+28(x-5)+M = 0

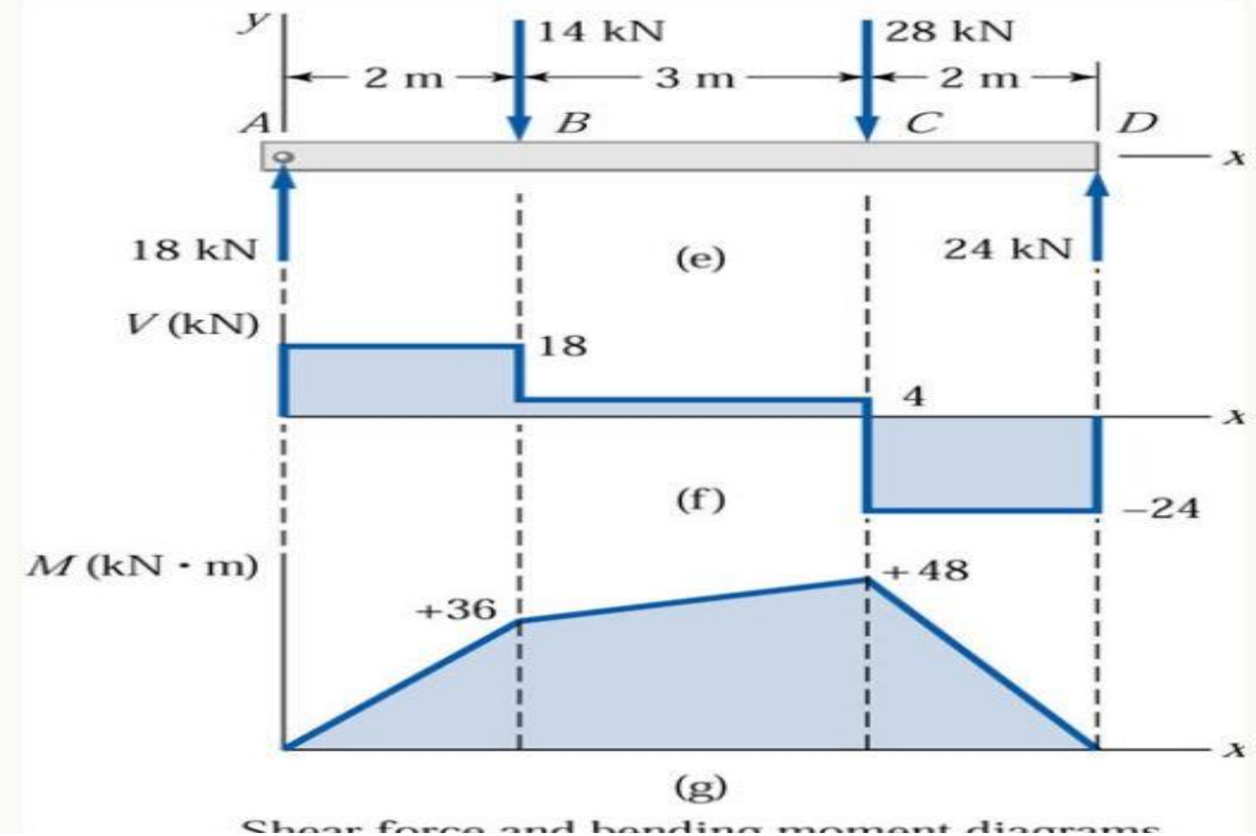
M = +18x-14(x-2) – (x-5)

<u>= -24x+168 kN· m</u>

Point G is at position 3 in fig a

Part 2

Shear force and bending moment diagram



Important points

- The V-diagram reveals that the largest shear force in the beam is -24 kN : segment CD
- The M-diagram reveals that the maximum bending moment is +48 kN·m : the 28-kN load at C.

Any Questions ?


THANK YOU

WELLCOME TO MY PRESENTATION

Presented By
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Work Shop Superintendent
(Tech/Mechanical)
Mechanical Technology
Mymensingh Polytechnic Institute

Subject : Strength of Materials

Sub:Code:67064

Chapter -08

- Understand the analysis of shear stress in beams.

After the end of this lesson students will able to;

- Explain shear stress at a section of beam.
- Express the deduction of the formula for shear stress.
- Identify the distribution of shear stress across the section.
- Calculate shear stress in rectangular, triangular, circular and simple composite sections.
- Solve problems related to shear stress

SHEAR STRESSES IN BEAMS

Introduction:

In the earlier chapter, the variation of bending stress across a beam section was studied. The bending stress is due to bending moment at the section.

The bending stress act longitudinally and its intensity is directly proportional to its distance from neutral axis.

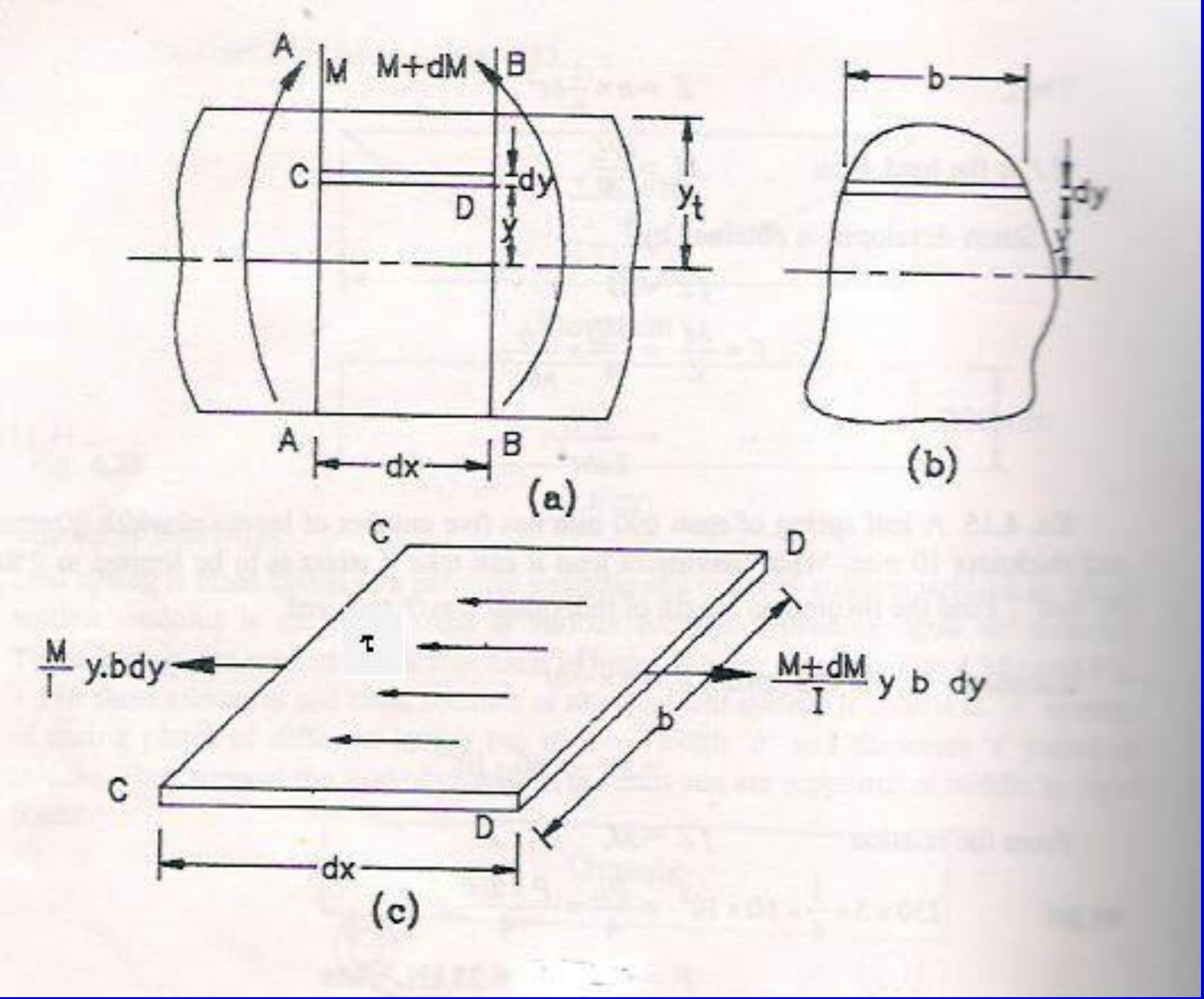
A typical beam section is subjected to shear force in addition to bending moment. The variation of shearing stress, which is due to the presence of shear force, is studied in this chapter.

SHEAR STRESSES IN BEAMS

The stresses induced by shear force at a section in a beam may be analyzed as follows:

Consider an elemental length of a beam between the sections AA and BB separated by a distance dx, as shown in the following figure. Let the moments acting at AA and BB be M and M+ dM respectively.

Let CD be a fibre of thickness dy at a distance y from the neutral axis. Then bending stress at left side of the fibre CD = $\frac{M y}{I}$


A
M
M+dM
B
C
dy
D
y
y_t
A
B
dx
(a)
b
dy
y
(b)
C
D
τ
$\frac{M}{I}$ y.bdy
$\frac{M+dM}{I}$ y b dy
b
C
D
dx
(c)

Force on the left side of the layer CD = $\frac{M\,y\,(b.dy)}{I}$

and force on the right side of the layer CD = $\frac{(M+dM).y.(b.dy)}{I}$

Therefore unbalanced force, towards right, on the layer CD

is = $\frac{dM.y.b.dy}{I}$

There are a number of such elements above the section CD.

Hence the unbalanced horizontal force above the section CD =

$$\int_{y}^{y_t} \frac{dM.y.b.dy}{I}$$

This horizontal force is resisted by the resisting force provided by shearing stresses acting horizontally on the plane at CD.

Let the intensity of shear stress be τ. Equating the resisting force provided by the shearing stress to the unbalanced horizontal force we have:

$$\tau \, .b.dx = \int_{y}^{y_t} \frac{dM}{I} .y \,.\, b.\, dy$$

$$\text{or} \quad \tau = \frac{dM}{dx} \cdot \frac{1}{I.b} \int_{y}^{y_t} y.da$$

where da = b.dy is area of the element .

where the term $\int_{y}^{y_t} y.da = a\bar{y}$ = Moment of, area above the fibre CD about the NA.

but the term dM / dx = F, the shear force. Substituting in the expression for τ, we obtain :

$$\tau = \frac{F\, a\, \bar{y}}{I\, b}$$

where :

F = shear force at a section in a beam

a = area above or below a fibre (shaded area)

$\bar{y}$ = dist. from N.A. to the centroid of the shaded area

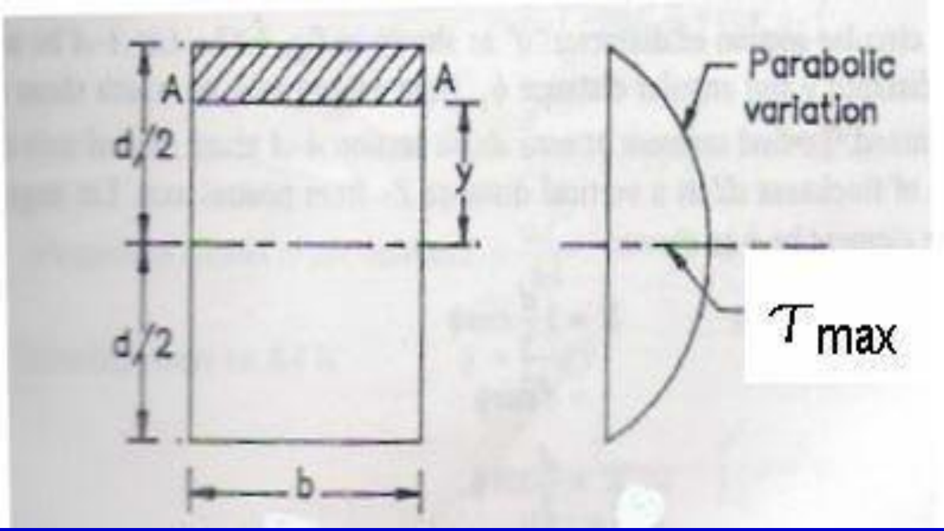
I = M.I. of the entire section about the N.A

b = breadth of the fibre.

Note *:The above expression is for horizontal shear stress. From The principle of complementary shear, this horizontal shear stress Is accompanied by a vertical shear stress of the same intensity.*

Shear stress variation across a few standard cross sections

1. Rectangular section:



Consider a rectangular section of width b and depth d subjected to shearing force F. Let AA be a fibre at a distance y from the neutral axis as shown in fig.

From the equation for shear stress :

$$\tau = \frac{F a \bar{y}}{I b}$$

where : $a = b\left[\frac{d}{2} - y\right]$

$$\bar{y} = y + 1/2\left[\frac{d}{2} \quad y\right] = y + \frac{1}{2}\left[\frac{d}{2}\right] - \frac{y}{2} = \frac{1}{2}\left[\frac{d}{2}\right] + \frac{y}{2} = 1/2\left[\frac{d}{2} + y\right]$$

and $I = bd^3/12$

substituting, $$\tau = \frac{F \,.\, b\left[\frac{d}{2} \quad y\right] . \, 1/2\left[\frac{d}{2} + y\right]}{(bd^3/12) \,.\, b}$$

$$= \frac{6F\left[\frac{d}{2} - y\right] \times \left[\frac{d}{2} + y\right]}{bd^3} = \frac{6F}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$

$$\tau = \frac{6F}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$

Thus the shear stress variation is parabolic. When,

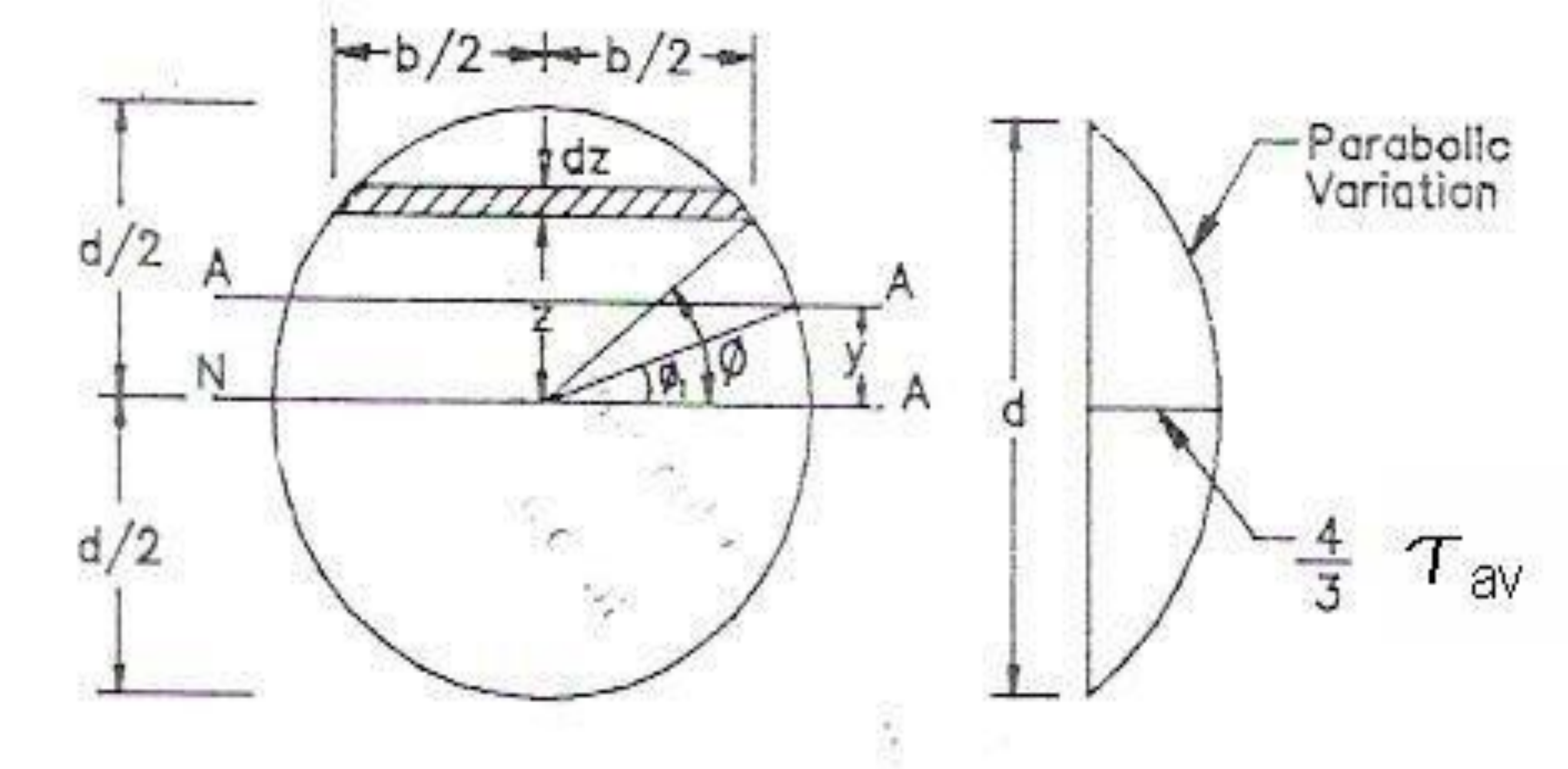
i) at $y = d/2$ $\iota = 0$

ii) at $y = - d/2$ $\iota = 0$

(iii) $y = 0$, τ is maximum and its value is $= \dfrac{6\,Fd^2}{4\,bd^3} = \dfrac{1.5\,F}{bd}$

$$= \frac{1.5\ \textit{Shear Force}}{\textit{Area}}$$

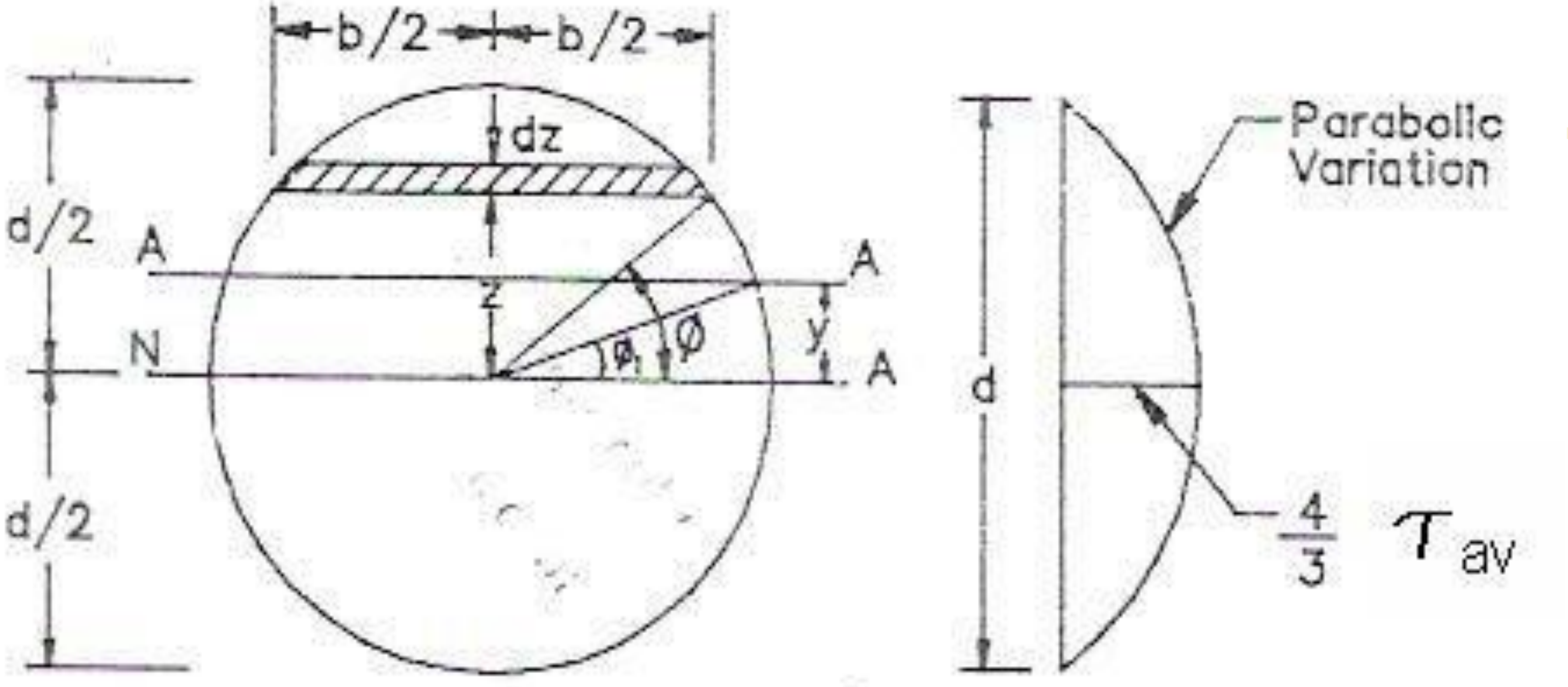
that is, $\tau_{max} = 1.5\,\tau_{avg}$, for a rectangular section, and this occurs at the neutral axis. Where τ_{avg} = Shear force/ Area



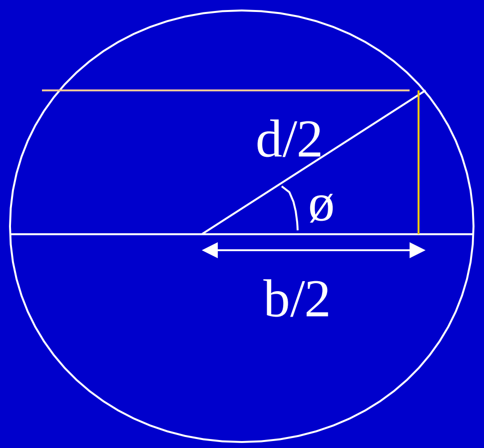
2. Circular section:

Consider a circular section of diameter d , as shown in fig. Let AA be a fibre at a vertical distance y and angle Φ_1,from N.A.,on which shear stress is to be determined.

To find moment of area above the fibre AA about the N.A., consider an element of thickness dz at a vertical distance z from the N.A. Let the angular distance of the element be Φ, as shown in fig.

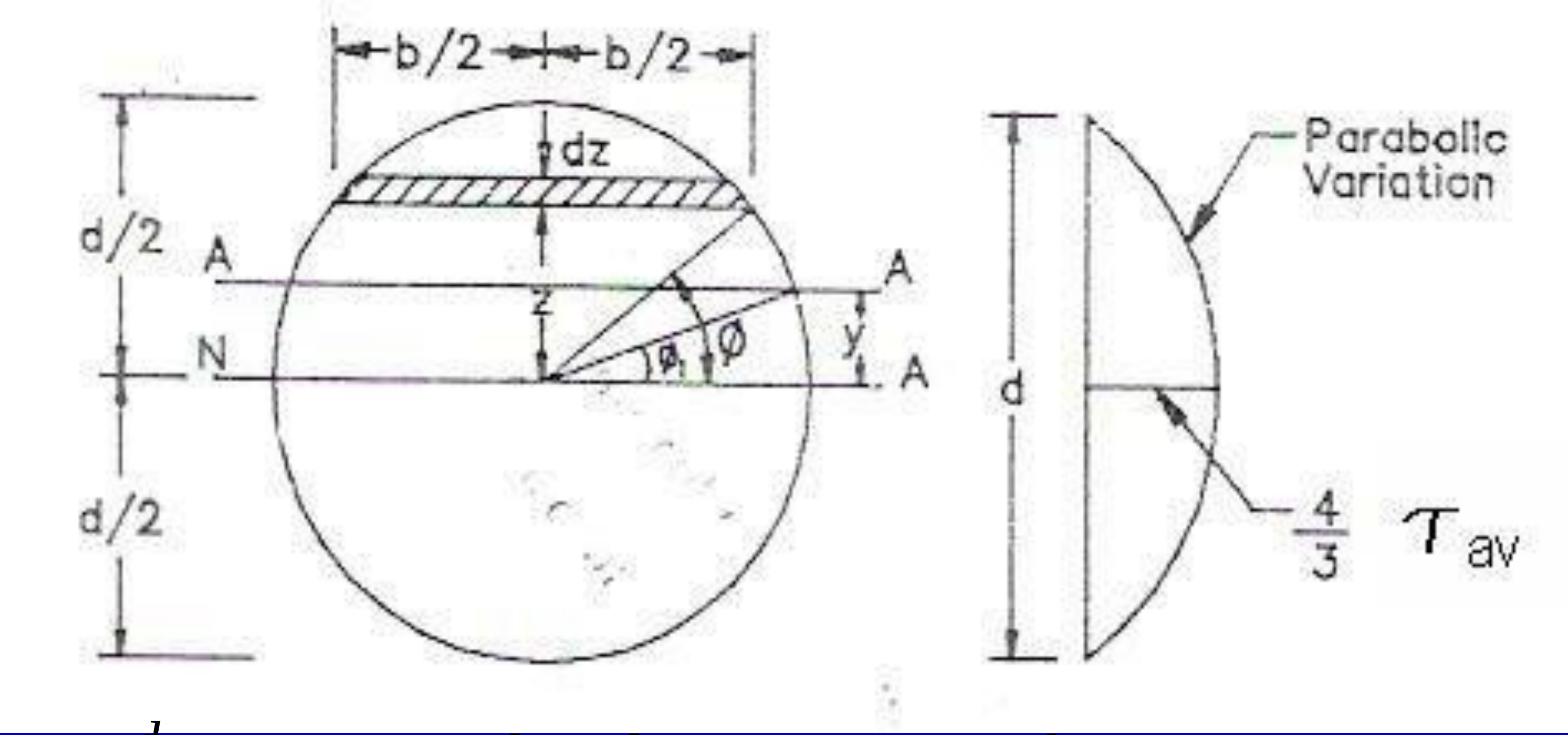


area of the element, A = b dz



$$\cos ø = \frac{b/2}{d/2}$$

Width of the element , b = d cos ø



$$z = \frac{d}{2}\sin\phi \quad \longrightarrow \quad \frac{dz}{d\phi} = \frac{d}{2}\cos\phi \quad \longrightarrow \quad dz = \frac{d}{2}\cos\phi\, d\phi$$

area of the element, A = b dz

$$A = d\cos\phi\left(\frac{d}{2}\right)\cos\phi\, d\phi = \frac{d^2\cos^2\phi\, d\phi}{2}$$

Moment of area of the element about the N.A.= area× z

$$= \frac{d^2}{2} \cos^2\phi \, d\phi \, \frac{d}{2} \sin\phi$$

$$= \frac{d^3}{4} \cos^2\phi \, \sin\phi \, d\phi$$

Therefore moment of the entire area ,above the fibre AA, about the N.A.= a $\bar{y}$

$$= \int_{\phi 1}^{\pi/2} \frac{d^3}{4} \cos^2\phi \, \sin\phi \, d\phi$$

if cosø = t , dt/dø = - sin ø , dt = - sin ø dø and $-t^3/3$ is integration

$$= \frac{d^3}{4} \left[\frac{-\cos^3\phi}{3} \right]_{\phi_1}^{\pi/2}$$

$$= \frac{d^3}{12} [-\cos^3\frac{\pi}{2} + \cos^3\phi_1]$$

$$= \frac{d^3}{12} [\cos^3\phi_1]$$

Moment of inertia of the section , $I = \pi d^4 / 64$

Substituting in the expression for shear stress,

$$\tau = \frac{F a \bar{y}}{I b}$$

$$\tau = \frac{F \dfrac{d^3}{12} \cos^3 \phi_1}{\dfrac{\pi d^4}{64} d \cos \phi_1} = \frac{16}{3} \times \frac{F}{\pi d^2} \cos^2 \phi_1$$

$$= \frac{16}{3} \times \frac{F}{\pi d^2} (1 - \sin^2 \phi_1)$$

Where b = width of the fibre AA = d cos Φ_1

$$= \frac{16\,F}{3\,\pi d^2}\left[1-\left(\frac{y}{d/2}\right)^2\right]$$

Hence shear stress varies parabolically over the depth.
Its value is zero at the extreme fibres where y = d / 2 and its value is maximum when y = 0 (at the N.A.) and is given by :

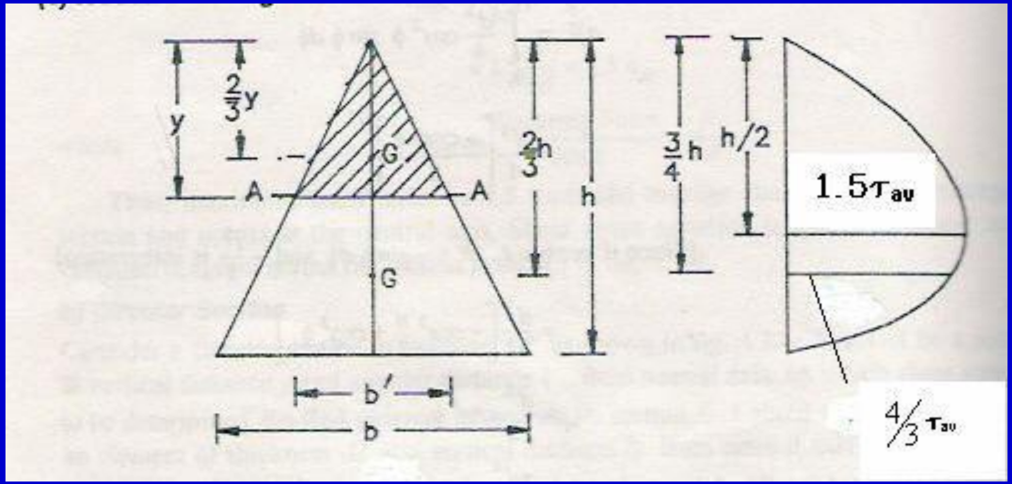
$$\tau_{max} = \frac{16\,F}{3\,\pi d^2}$$

$$= \frac{4\,F}{3\,\pi d^2/4} \qquad = \frac{4\,F}{3\,A}$$

$$\frac{F}{A} = \frac{\text{Shear Force}}{\text{Area of cross section}} = \text{Average shear stress}$$

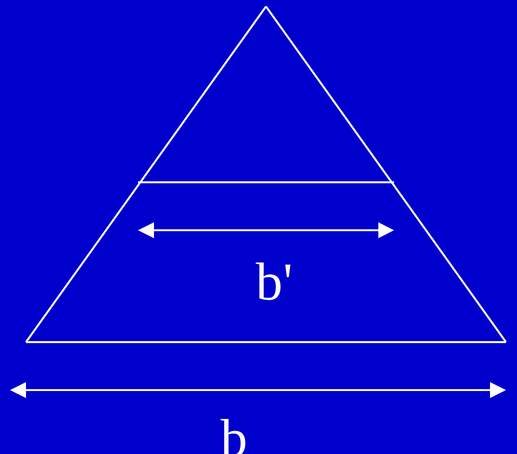
Thus in circular sections shear stress is maximum at the centre, and is equal to 4 / 3 times the average shear stress.

3. Isosceles Triangular section :



Let AA be a fibre at a distance y from the top.

Shear stress in general , $\tau = \frac{F\,a\,\bar{y}}{I\,b}$



$$\tau = F\left\{\frac{\left(\frac{1}{2}b'y\right)\left(\frac{2h}{3}-\frac{2y}{3}\right)}{\frac{bh^3}{36}\;b'}\right\} \qquad \text{—— } 1$$

$$\tau = \frac{12\,F\;y\,(h - y)}{bh^3}$$

i) at $y = 0$ $\tau = 0$

ii) at $y = h$ $\tau = 0$

At the centroid , y = 2h / 3 , substituting in equation 1,

$$\tau = \frac{12\,F}{bh^3}\;2h/3\,(h - 2h/3) = \frac{8\,F}{3\,bh}$$

4 Shear Force

$$\text{or } \tau = \frac{4F}{3\frac{bh}{2}} = \frac{\quad}{3\ \text{Area}} = \frac{4}{3}\tau_{avg}\text{, at the N.A.}$$

For shear stress ,τ, to be max., $\frac{d\tau}{dy} = 0$,

$$\frac{12F}{bh^3}(h - 2y) = 0$$

or $y = h / 2$, substituting in the expression for τ,

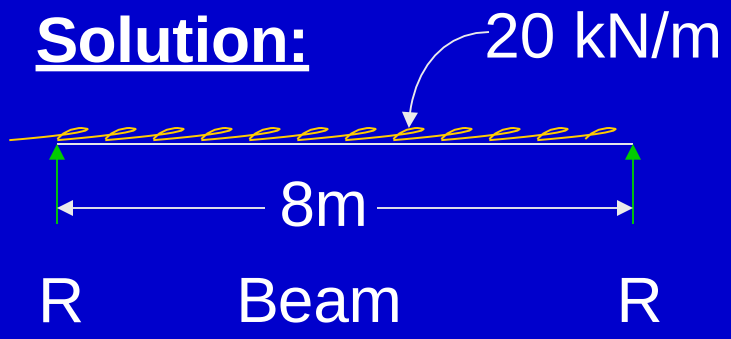
$$\tau_{max} = \frac{12F}{bh^3}\,\frac{h}{2}\left(h - \frac{h}{2}\right) = \frac{3F}{bh} = \frac{1.5F}{bh/2} = 1.5\,\tau_{avg}$$

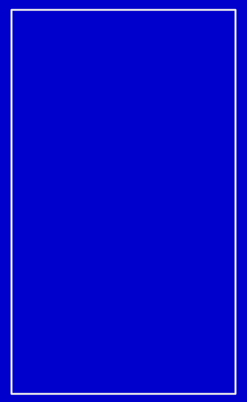
Thus, max. shear stress occurs at half the depth and its value is 1.5 times the average shear stress, in the case of an isosceles triangle.

SOLVED PROBLEMS

1. A s. s. beam of span 8m carries a UDL of 20 kN/m over its entire span. The c / s of the beam is a rectangle 120mm × 180mm deep. Draw the shear stress distribution at 1m from the left support, by considering horizontal fibres 30mm apart from top to bottom in the cross section.

Solution:





c/s of beam

$$R = \frac{20 \times 8}{2} = 80 \text{ kN}$$

Shear stress at a horizontal fibre in a beam c/s is given by:

$$\tau = \frac{F\,a\,\bar{y}}{I\,b}$$

Here, F = Shear Force at 1m from left support

= + R – 20×1 = 80 – 20 = 60kN

I = M.I.of the entire section about N.A. = $bd^3/12$

= $120 \times 180^3 / 12 = 58.3 \times 10^6$ mm^4

b = breadth of the fibre

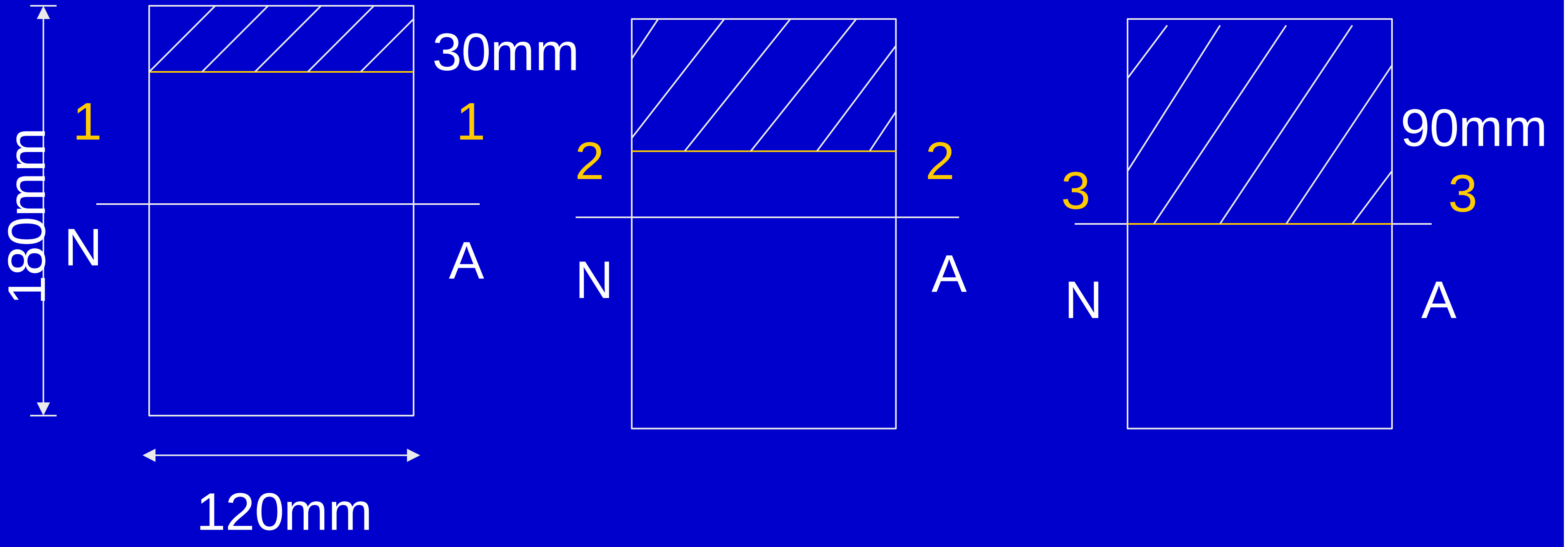
Note:

a = area **above or below** the fibre under consideration (shaded area)

$\bar{y}$ = distance from N.A. to the centroid of the shaded area

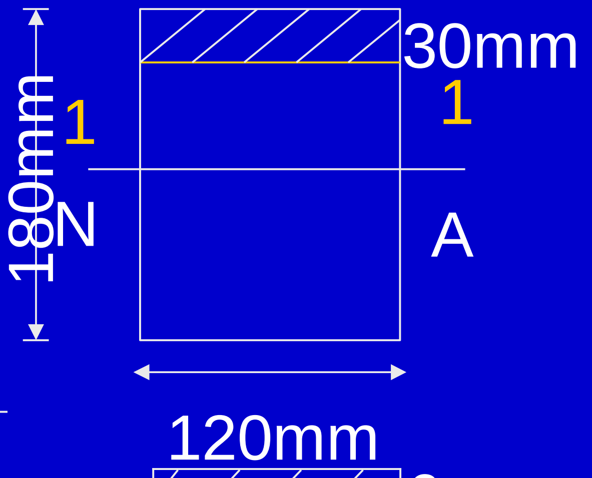
Shear stress values at fibres 30mm apart, starting from top

$\tau_{topmost} = \tau_{bottommost} = 0$,



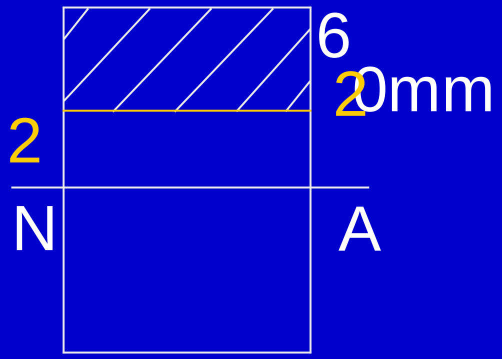
τ at 30mm from top (fibre 1-1) = $\dfrac{F a \bar{y}}{}$

$= \dfrac{60 \times 10^3 (120 \times 30) 75}{58.3 \times 10^6 \times 120} = 2.3156\ N/mm^2$



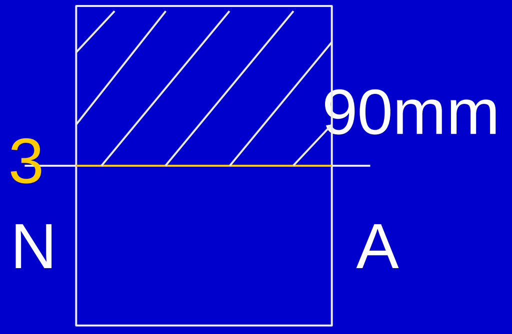
τ at 60mm from top (fibre 2-2) = $\dfrac{F a \bar{y}}{I\, b}$

$= \dfrac{60 \times 10^3 (120 \times 60) 60}{58.3 \times \times 10^6 \times 120} = 3.705\ N/mm^2$

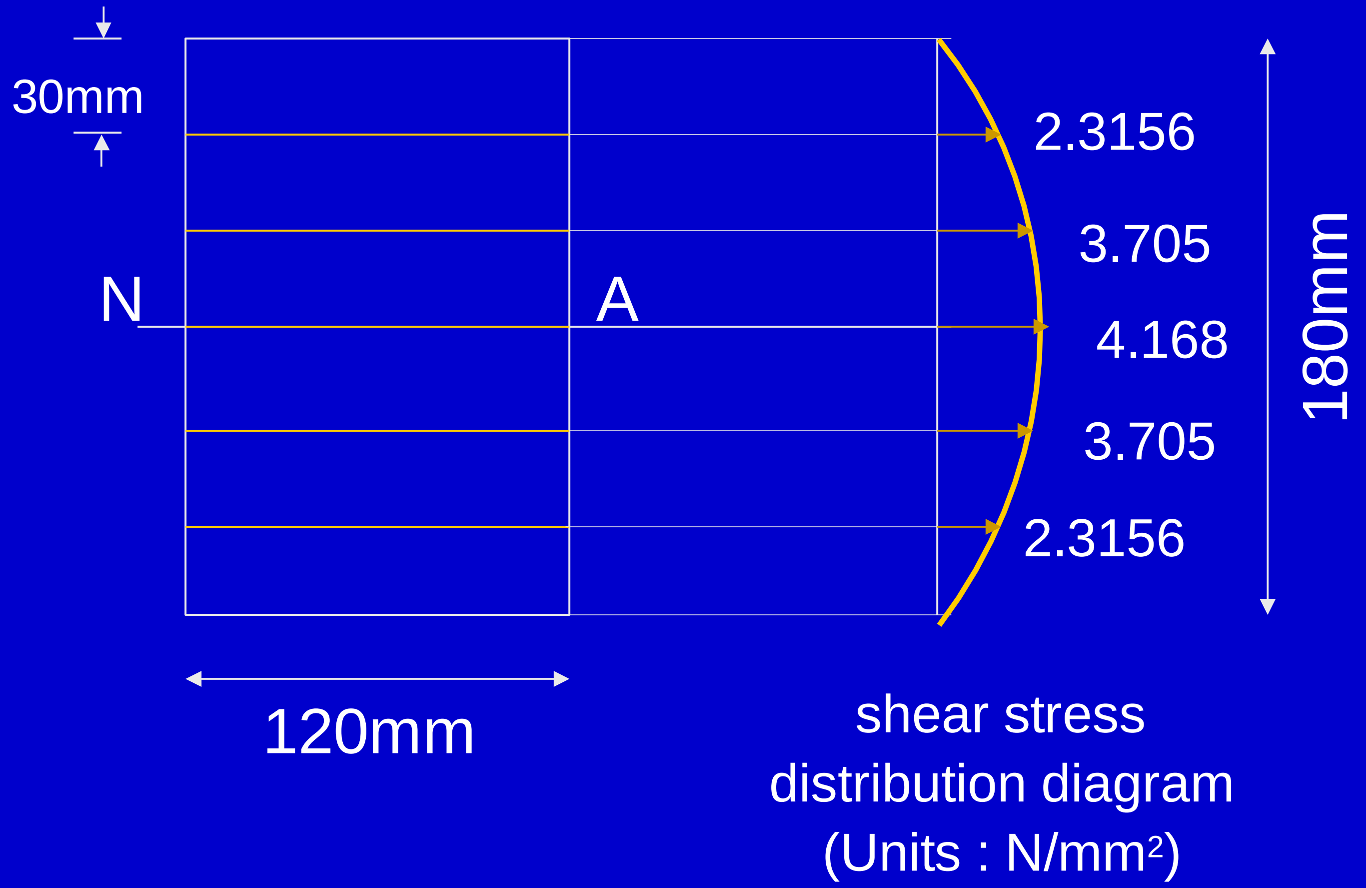


τ at 90mm from top (fibre 3-3) = $\dfrac{F a \bar{y}}{}$

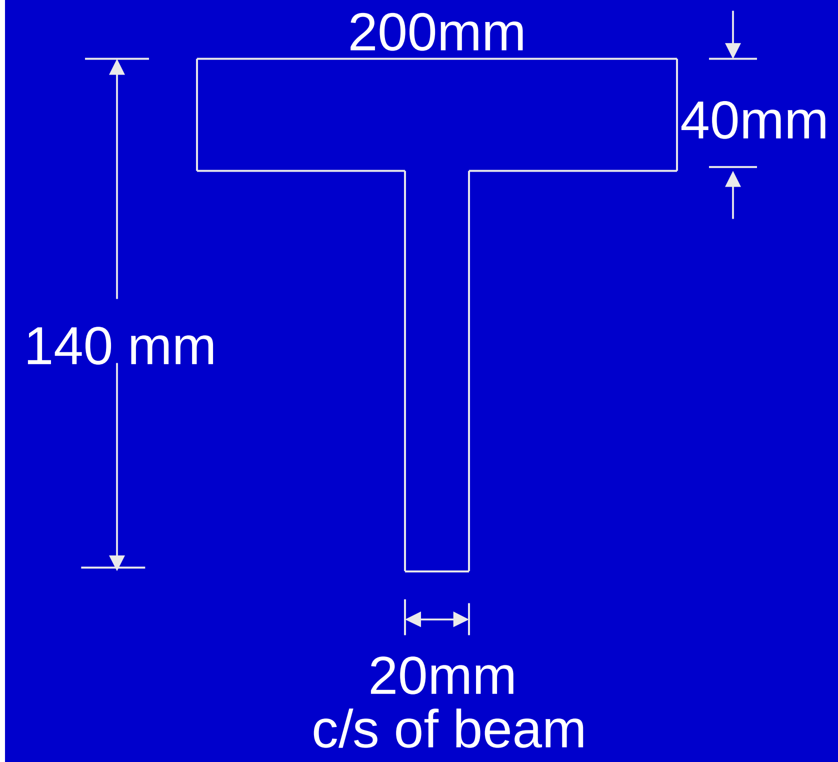
$= \dfrac{60 \times 10^3 (120 \times 90) 45}{58.3 \times 10^6 \times 120} = 4.168\ N/mm^2$



Due to symmetry of the section about the N.A., the corresponding fibres below the N.A. have the same stresses. The shear stress distribution diagram is as shown below.



2. The cross section of a beam is a T section of overall depth 140 mm, width of flange 200mm, thickness of flange 40mm and thickness of web 20mm.Draw the shear stress distribution diagram if it carries a shear force of 60 kN.

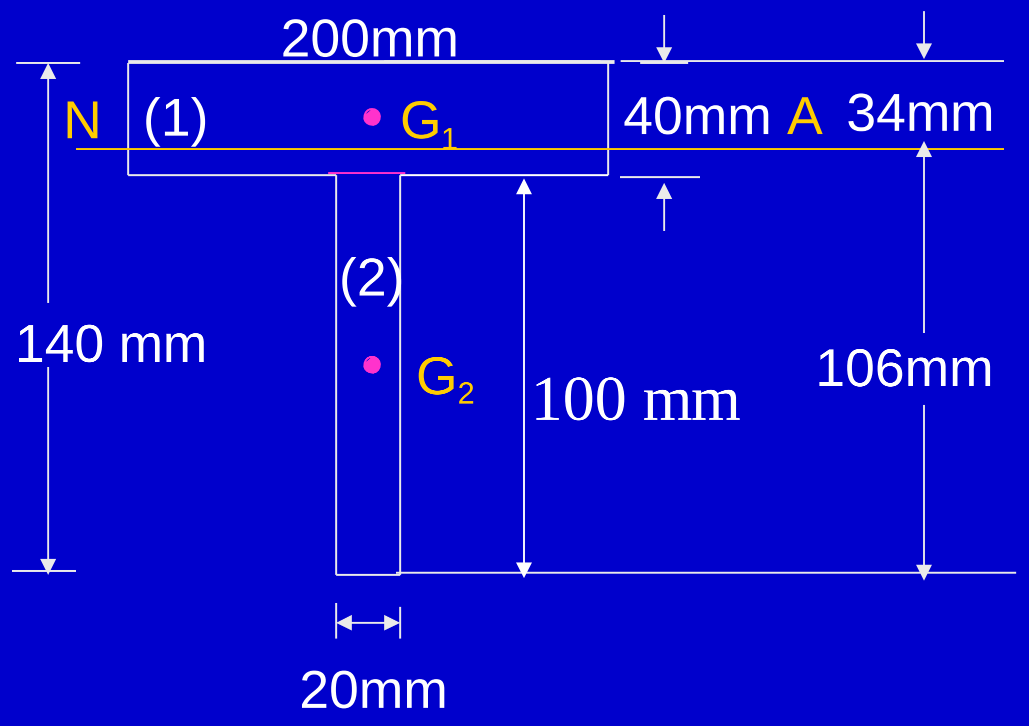


Solution:

To locate the N.A.

$$\bar{y} = \frac{(200 \times 40 \times 120) + (100 \times 20 \times 50)}{200 \times 40 + 100 \times 20}$$

= 106mm from bottom



To find M.I. of the section about the N.A.

$$I = I_{NA(1)} + I_{NA(2)} = \left[(200 \times 40^3)/12 + (200 \times 40)(34-20)^2 \right] + \left[20 \times 100^3 / 12 + (20 \times 100)(106-50)^2 \right]$$

$$= 10.57 \times 10^6 \ mm^4$$

Shear stress values at salient fibres :

- To draw the shear stress variation diagram shear stress values are obtained at a few significant (salient) fibres which are as under :

  (i) top most and bottom most fibres(where shear stress is always = 0)

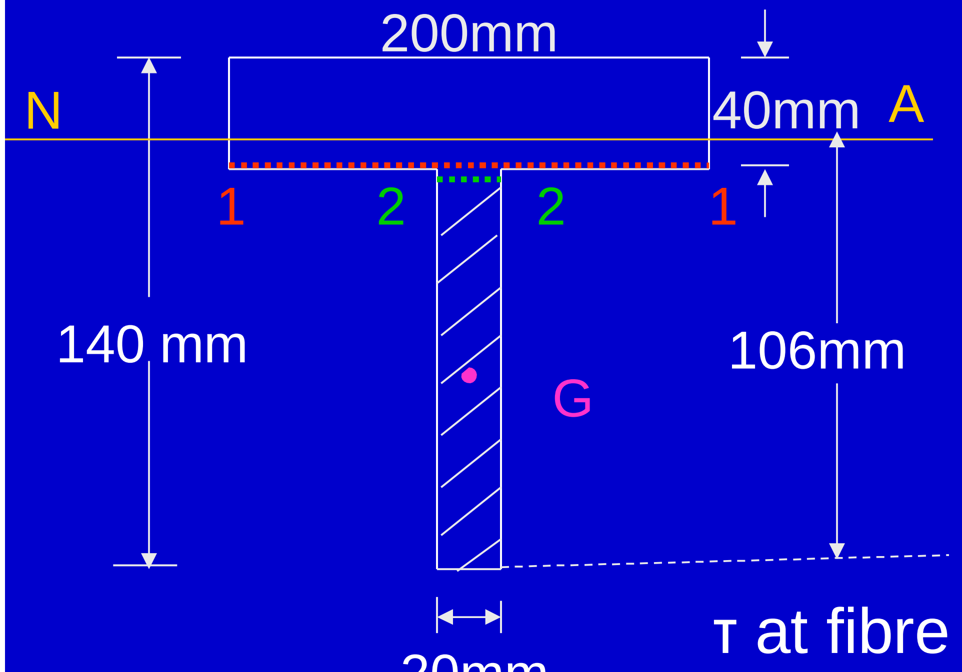
  (ii) at the N.A.

  (iii) two adjacent fibres, at the junction of flange and web, one fibre just in flange and the other fibre just in web.

$$\tau \text{ at the N.A.} = \frac{F a \bar{y}}{I b} = \frac{(60 \times 10^3)(200 \times 34)(34/2)}{(10.57 \times 10^6)(200)}$$

$$= 3.28 \text{ N / mm}^2$$

τ at the junction of flange and web :



c/s of beam

Consider two adjacent fibres 1-1 and 2-2 as shown in fig.

$$\tau \text{ at fibre 1-1} = \frac{60 \times 10^3\,(200 \times 40)(34-20)}{(10.57 \times 10^6)(200)}$$

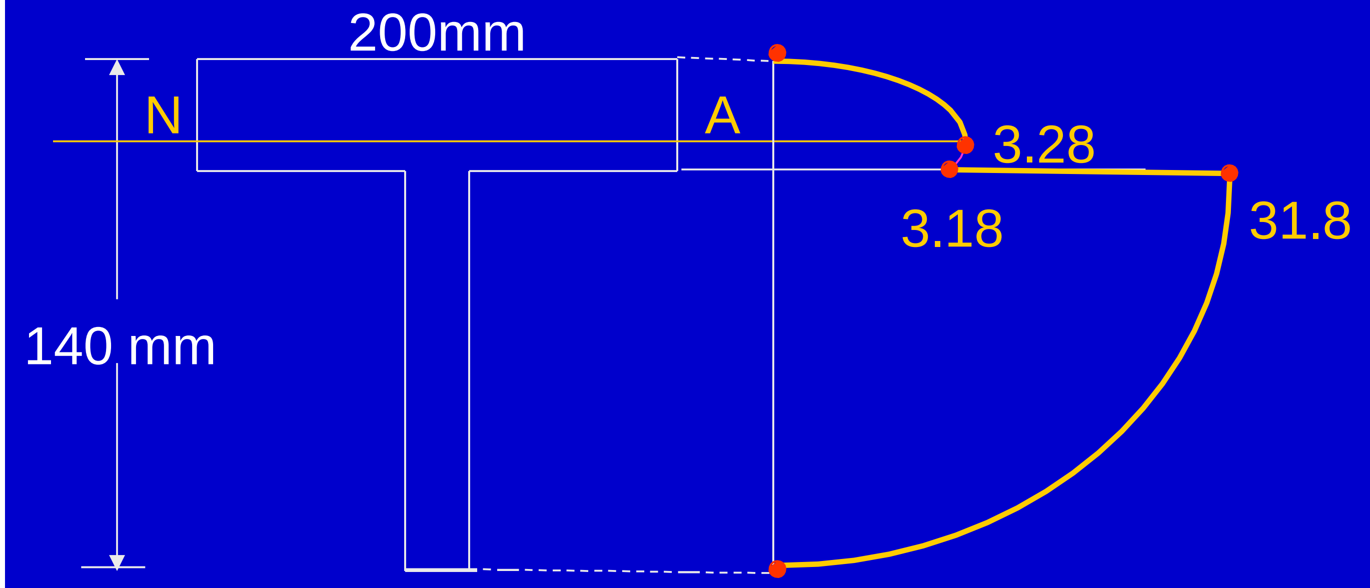
area above the fibre under consideration

$$= 3.18 \text{ N/mm}^2$$

or **area below the fibre under consideration**

$$\tau \text{ at fibre 1-1} = \frac{60 \times 10^3\,(100 \times 20)(106-50)}{(10.57 \times 10^6)(200)} = 3.18 \text{ N/mm}^2$$

$$\tau \text{ at fibre 2-2} = \frac{60 \times 10^3 \,(100 \times 20)(106-50)}{(10.57 \times 10^6)(20)} = 31.8 \text{ N/mm}^2$$

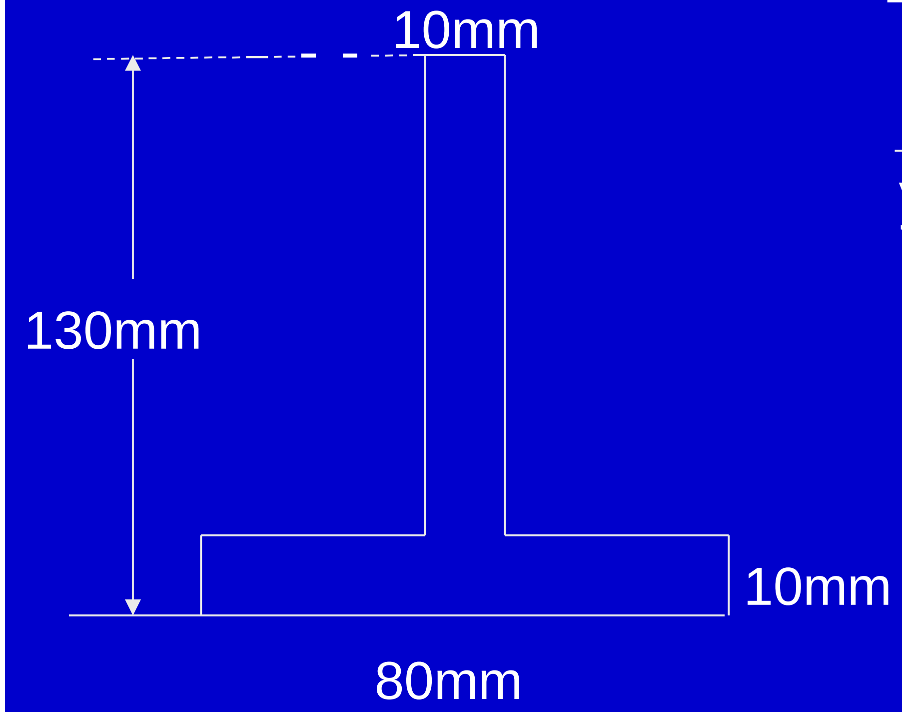


c/s of beam

shear stress distribution diagram (Units : N/mm²)

3. An inverted T section has an overall depth of 130mm,width and thickness of flange 80mm and 10mm respectively and thickness of web 10mm. Draw the shear stress distribution diagram across its c/s if it carries a shear force of 90 kN.

Solution :

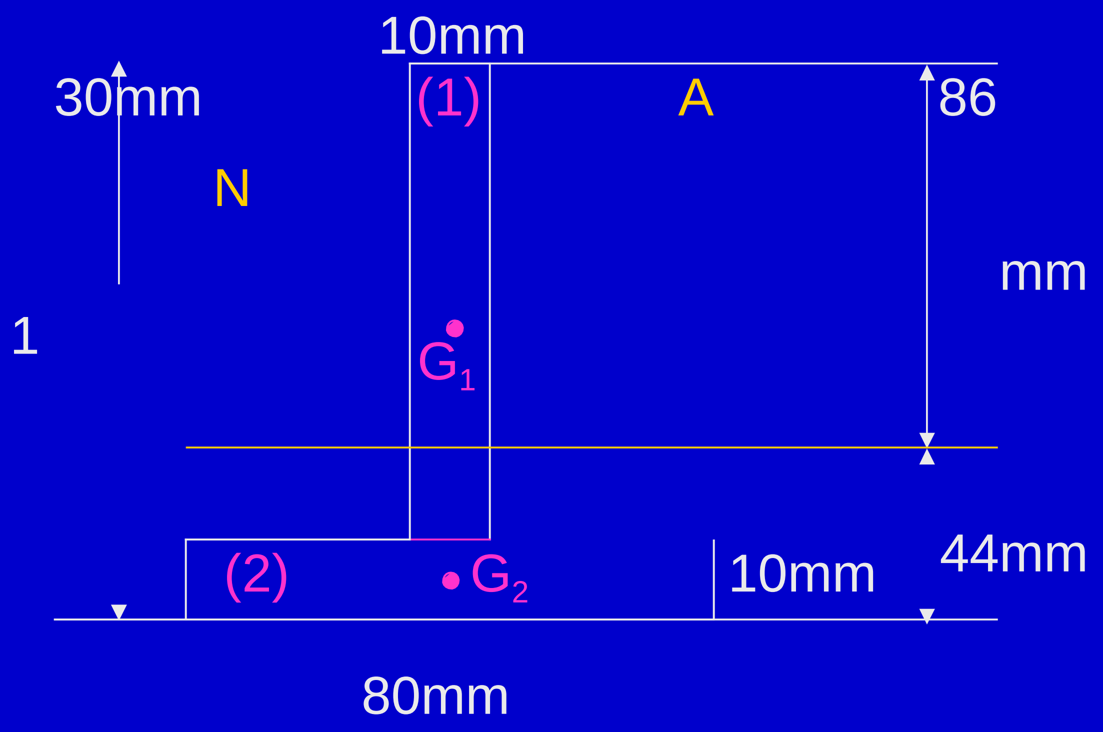


c/s of beam

To locate the N.A.

$$\bar{y} = \frac{(120 \times 10 \times 70) + (80 \times 10 \times 5)}{120 \times 10 + 80 \times 10}$$

= 44 mm from bottom



To find M.I. of the section about the N.A.

$$I = I_{NA\,(1)} + I_{NA\,(2)} = \left[(10 \times 120^3)/12 + (10 \times 120)(86-60)^2 \right]$$

$$+ \left[(80 \times 10^3)/12 + (80 \times 10)(44-5)^2 \right]$$

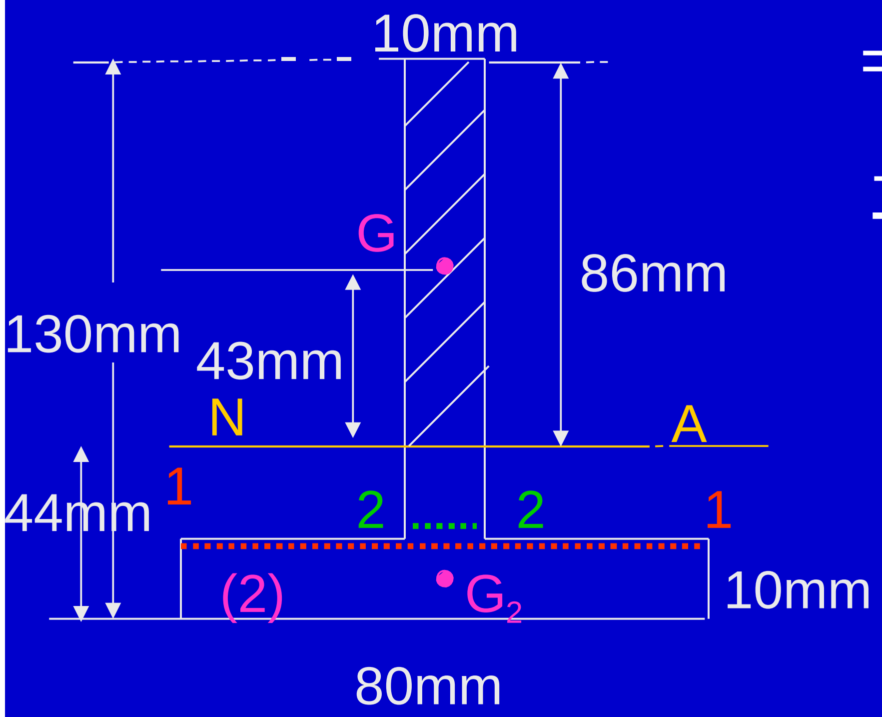
$$= 3.475 \times 10^6 \ mm^4$$

Shear stress values at salient fibres :

(i) At top most and bottom most fibres shear stress is always = 0

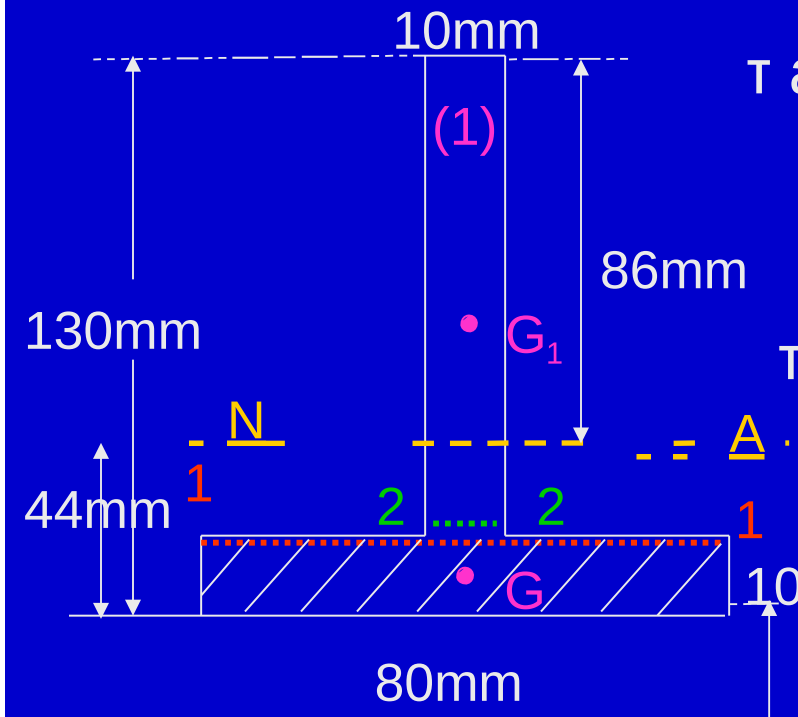
(ii) τ at the N.A.= $\dfrac{F a \bar{y}}{I b}$ $= \dfrac{(90 \times 10^3)(86 \times 10)(43)}{(3.475 \times 10^6)(10)}$

$= 95.78 \text{ N / mm}^2$



τ at the junction of flange and web :

Consider two adjacent fibres 1-1 and 2-2 as shown in fig.

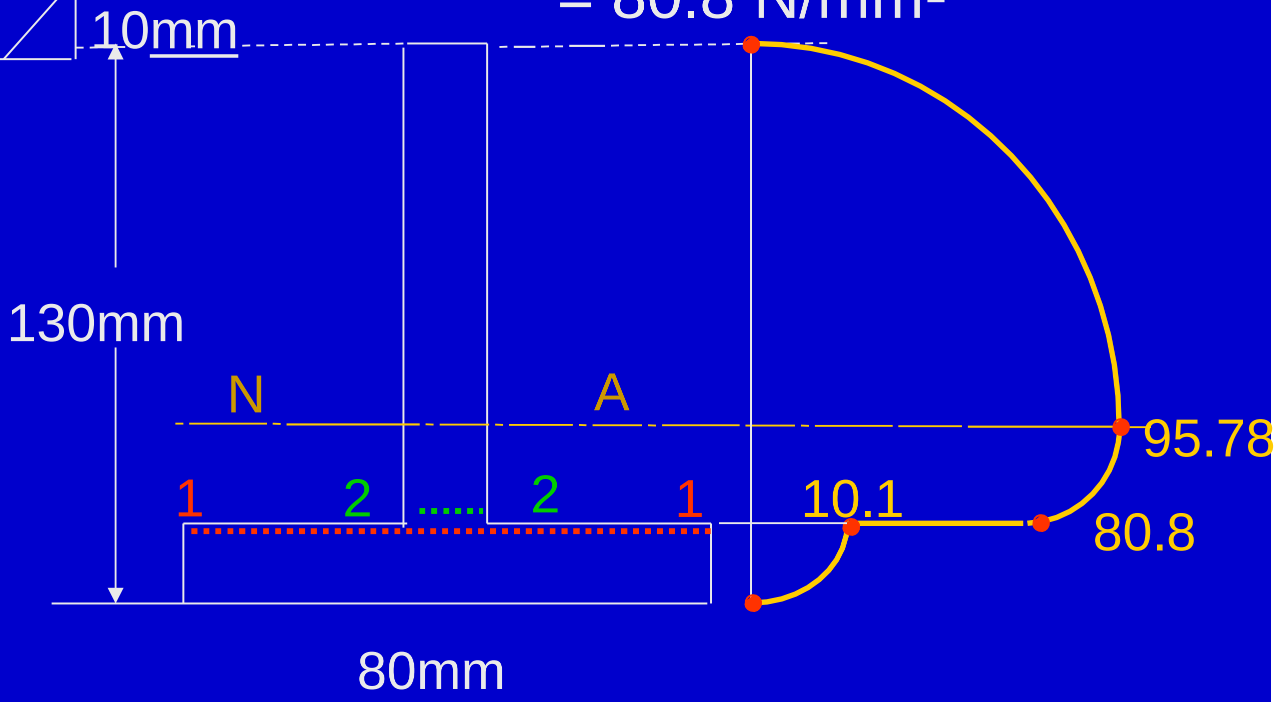


$$\tau \text{ at fibre } 1\text{-}1 = \frac{90 \times 10^3 (80 \times 10)(44-5)}{(3.475 \times 10^6)(80)}$$

$$= 10.1 \text{ N/mm}^2$$

$$\tau \text{ at fibre } 2\text{-}2 = \frac{90 \times 10^3 (80 \times 10)(44-5)}{(3.475 \times 10^6)(10)}$$

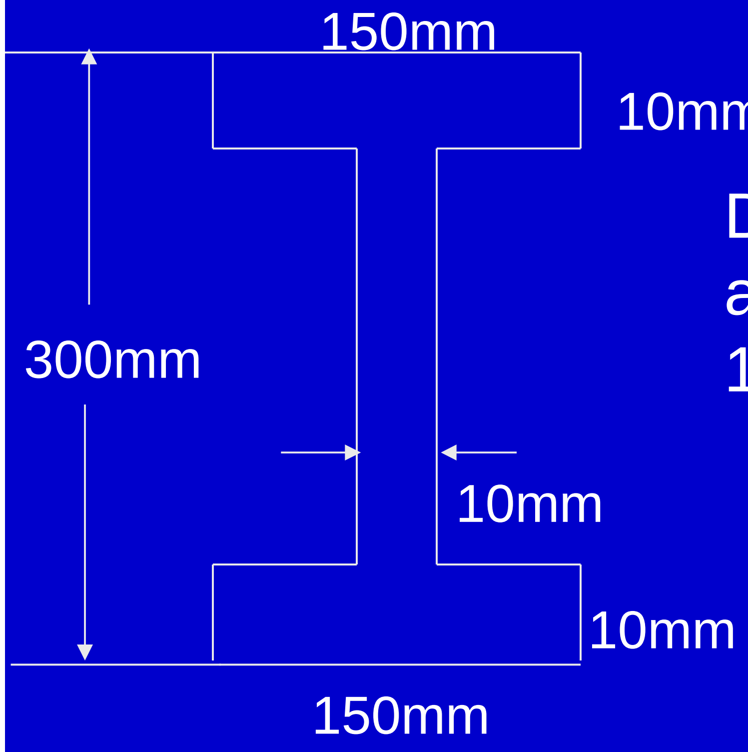
$$= 80.8 \text{ N/mm}^2$$



shear stress
distribution diagram

(Units : N/mm²)

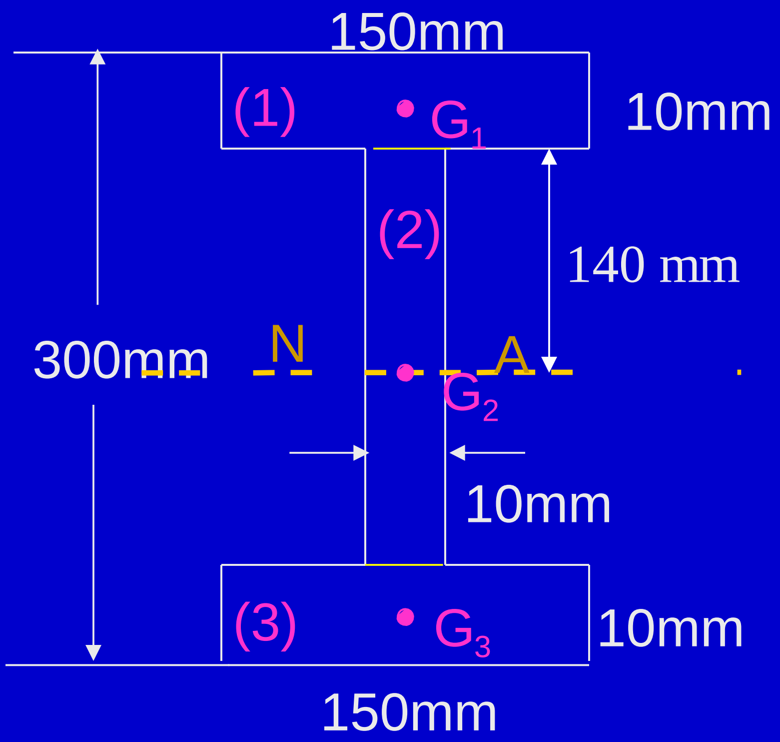
4. The c/s of a beam is an I–section as shown in fig. Draw the shear stress variation diagram if it carries a shear force of 200kN.



Solution :

Due to symmetry about the centroidal x axis , the N.A.lies at half the depth,i.e., at 150mm from the top.

To find M.I. of the section about the N.A.



$$I = I_{NA\,(1)} + I_{NA\,(2)} + I_{NA\,(3)}$$

$$= 2\,I_{NA\,(1)} + I_{NA\,(2)}$$

$$= 2\left[150 \times 10^3/12 + 150 \times 10(145)^2\right]$$

$$+ \left[10 \times 280^3/12\right]$$

$$= 81.39 \times 10^6\ mm^4$$

Shear stress values at salient fibres :

(i) At top most and bottom most fibres shear stress is always = 0

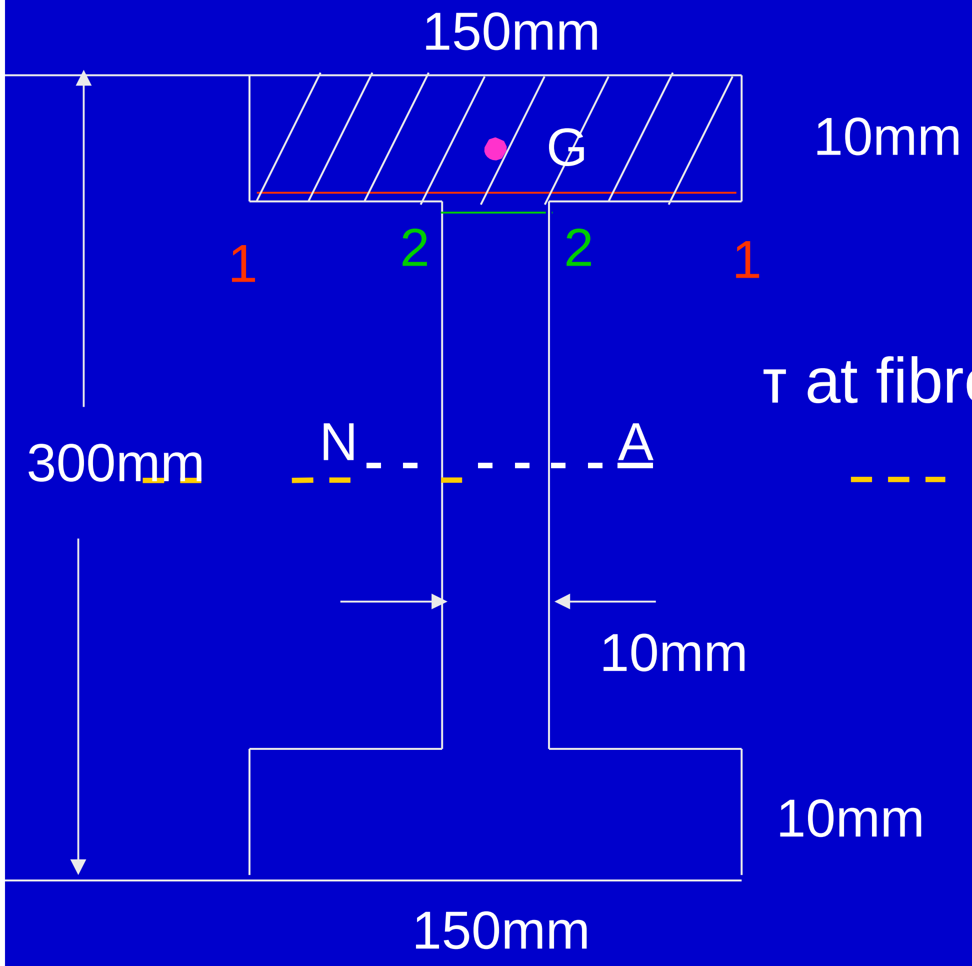
$$\tau \text{ at the N.A.} = \frac{F\, a\, \bar{y}}{I\, b}$$

$$= \frac{200 \times 10^3 \; [(150 \times 10)145 + (140 \times 10)70]}{81.39 \times 10^6 \; (10)}$$

$$= 77.52 \text{ N/mm}^2$$

τ at the junction of flange and web :

Consider two adjacent fibres 1-1 and 2-2 as shown in fig.
Due to symmetry of the section about the N.A., the shear stress variation diagram also is symmetric with corresponding fibres on either sides of the N.A.carrying the same shear stress.

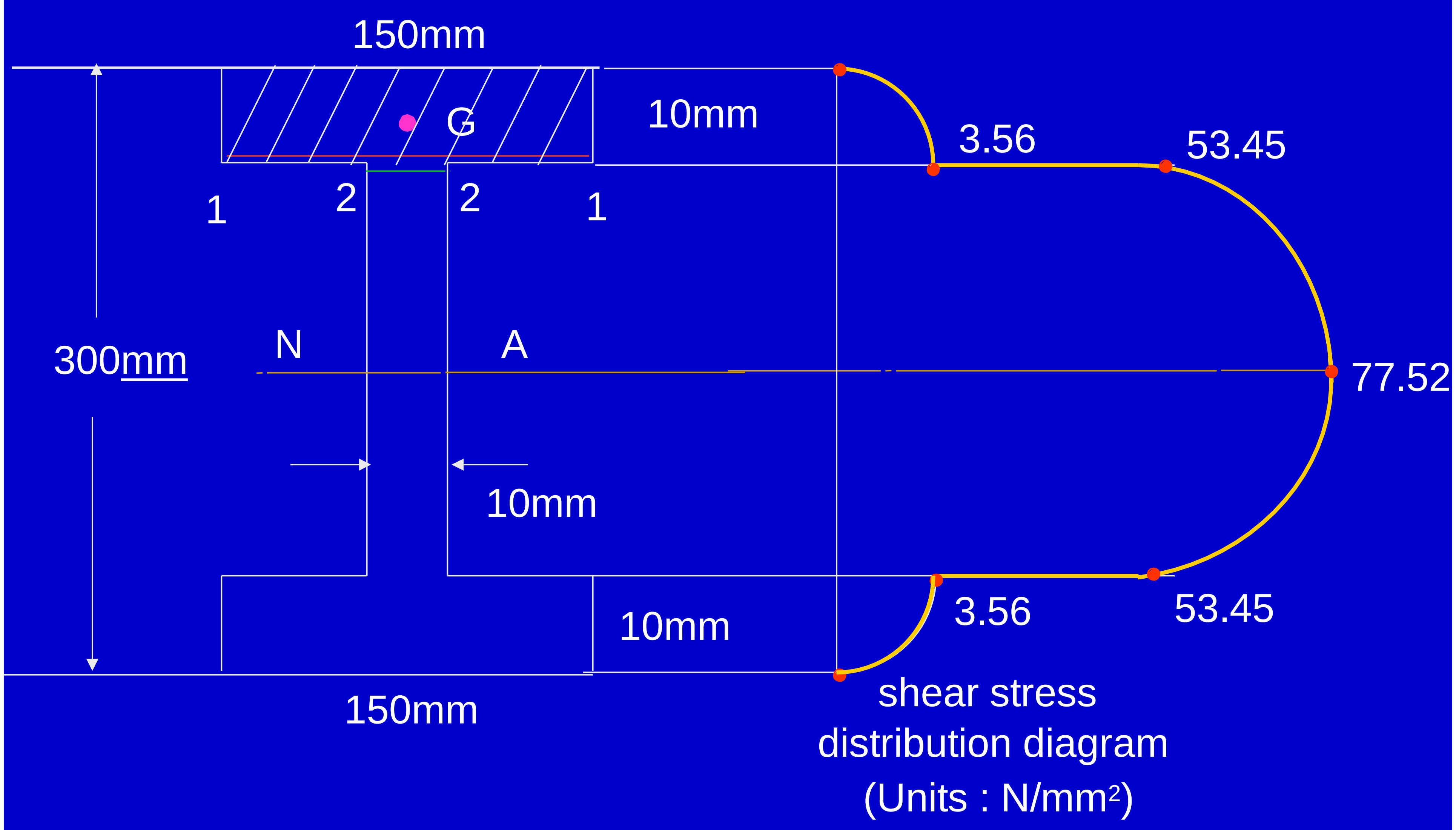


$$\tau \text{ at fibre 1-1} = \frac{200 \times 10^3 \,(150 \times 10)(145)}{(81.39 \times 10^6)(150)}$$

$$= 3.56 \text{ N/mm}^2$$

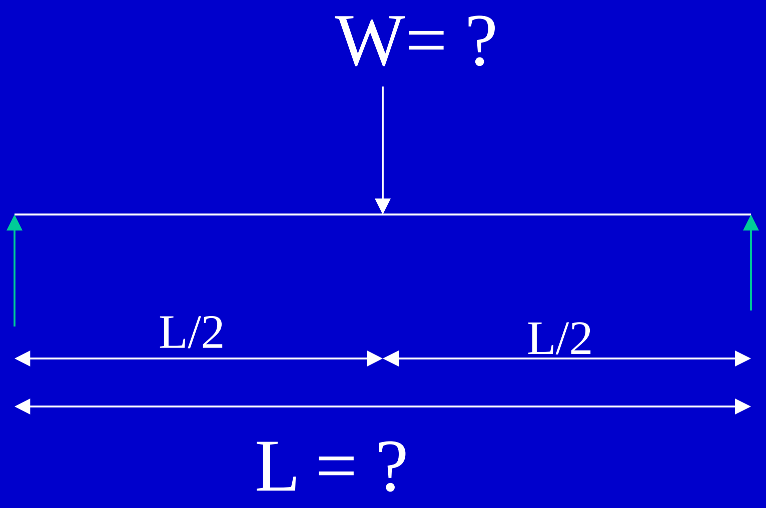
$$\tau \text{ at fibre 2-2} = \frac{200 \times 10^3 \,(150 \times 10)(145)}{(81.39 \times 10^6)(10)}$$

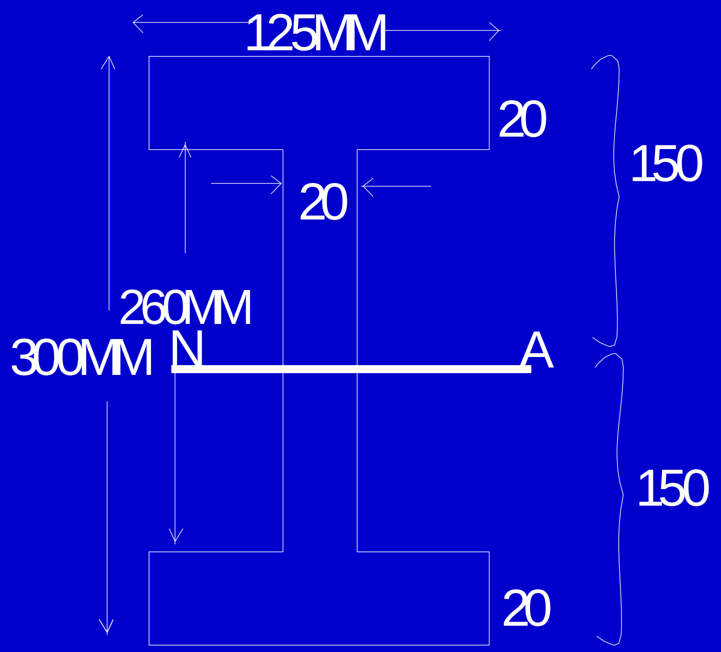
$$= 53.45 \text{ N/mm}^2$$



150mm
10mm
G
3.56
53.45
1
2
2
1
300mm
N
A
77.52
10mm
10mm
3.56
53.45
150mm
shear stress
distribution diagram
(Units : N/mm2)

(5) A Symmetrical I section beam rests freely on simple supports of span L . The beam carries a point load of W at the mid-span. The section of the beam is 125mm × 300 mm deep with flange and web thickness of 20 mm. If maximum bending stress and maximum shear stress are restricted to 150MPa and 45MPa respectively, calculate the values of L and W. $I_{NA} = 1.27 \times 10^8\ mm^4$





To calculate maximum shear stress

τ at the N.A.= $\dfrac{F\,a\,\bar{y}}{I\,b}$ Maximum Shear force F = W/2

$$\tau_{max} = \frac{W\,[(125\times20\times140) + (20\times130\times65)]}{(2 \times 20 \times I_{NA})} \quad \text{---------(1)}$$

But $\tau_{max} = 45 N/mm^2$, $I_{NA} = 1.27 \times 10^8\ mm^4$

Substituting in equation(1), *$W = 441.47\times10^3\ N = 441.47\ kN$*

To calculate maximum bending stress

Now, $\dfrac{M}{I_{NA}} = \dfrac{\sigma_b}{y}$

σ_b will be maximum when $y = y_{max}$ and $M = M_{max}$

$$M_{max} = WL/4 \quad \text{and} \quad y_{max} = 150\text{mm}$$

$$\Rightarrow \sigma_{b(max)} = \frac{M_{max} \times y_{max}}{I_{NA}}$$

$$\Rightarrow \sigma_{b(max)} = \frac{(WL/4) \times (150)}{I_{NA}} \qquad \text{-----(2)}$$

But $\sigma_{bmax} = 150\text{N/mm}^2$

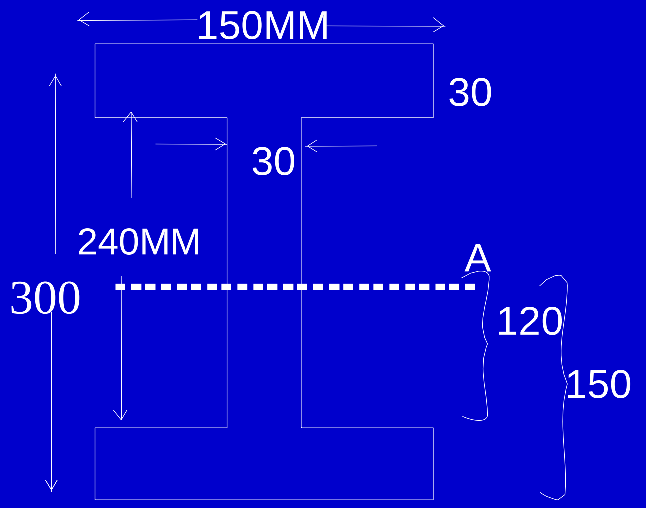
$W = 441.47 \times 10^3$ N

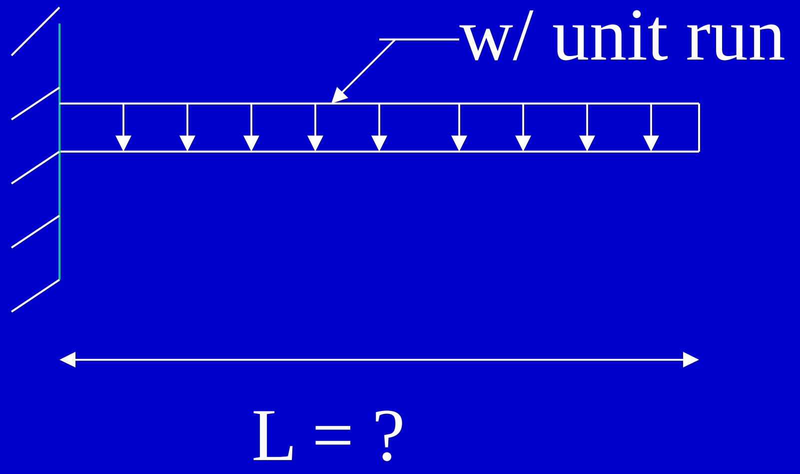
$I_{NA} = 1.27 \times 10^8$ mm^4

Substituting in equation (2)

L=1150 mm = 1.15 m

(6) A Cantilever beam of I section 150mm × 300 mm with a uniform thickness of flange and web 30mm carries an UDL throughout the span. Find the length of the beam if the max bending stress is 4 times the maximum shear stress.





To calculate maximum bending stress

Now, $$\frac{M}{I_{NA}} = \frac{\sigma_b}{y}$$

σ_b will be maximum when $y = y_{max}$ and $M = M_{max}$

The bending moment M will be maximum for the cantilever beam at the fixed end.

$M_{max} = w \times L \times L/2 = wL^2/2$ $\quad\quad$ $y_{max} = 150mm$

Now, $$\frac{M}{I_{NA}} = \frac{\sigma_b}{y} \quad \therefore \sigma_{b(max)} = \frac{(wL^2 \times 150)}{2I_{NA}} \quad \text{-----(1)}$$

To calculate maximum shear stress (occurs at N.A.)

τ at the N.A.= $\dfrac{F a \bar{y}}{I b}$ Max shear Force F = wL

$$\tau_{max} = \tau_{NA} = \frac{wL(150 \times 30 \times 135+120 \times 30 \times 60)}{30I_{NA}} \quad \text{----(2)}$$

It is given that $\sigma_{b(max)} = 4\,\tau_{max}$

$$\frac{150w\,L^2}{2I} = 4 \times \frac{wL(150 \times 30 \times 135 + 120 \times 30 \times 60)}{30I_{NA}}$$

$$\therefore\ L=1464mm =1.464m$$

EXERCISE PROBLEMS

1. Draw the shear stress variation diagram for a square section placed with one of its diagonals horizontal. Show that the maximum shear stress is equal to 9/8 times the average shear stress.

2. A timber beam 150mm x 250mm deep in c/s is simply supported at its ends and has a span of 3.5m.If the safe stress in bending is 7.5MPa find the maximum safe UDL the beam can carry. What is the maximum shear stress in the beam for the UDL calculated? (Ans: 7.66kN/m , 0.536N/mm^2)

3. The cross section of a beam is an isosceles triangle having base width 400mm and height 600mm. It is placed with its base horizontal and is subjected to a shear force of 90kN. Find the intensity of shear stress at the neutral axis. (Ans: 1 MPa)

4. A beam of channel section 120mm x 60mm has a uniform thickness of 15mm.Draw the shear stress diagram if it

   carries a shear force of 50kN.Find the ratio of maximum and mean shear stresses.
   (Ans:Shear stress values at significant fibres from bottom: 0, 6.67, 26.67, 35.24, 26.67, 6.67, 0 MPa. Ratio = 2.22)

5. The c/s of a beam is an unsymmetric I -section of overall depth 350mm, topflange 250mmx50mm,bottomflange 150mmx50mm, and web thickness 50mm. Draw the shear stress distribution diagram if it carries a shear force of

   80 kN.
   (Ans:Shear stress values at significant fibres from bottom: 0, 1.378, 4.134, 5.89, 5.06, 1.012, 0 MPa.)

6. A hollow rectangular box of outer dimensions 100mmx160mm deep and wall thickness 10mm carries a shear force of 150kN.

Draw the shear stress variation diagram.

(Ans: 0, 8.23, 20.59, 31.18, 20.59, 8.23,0)

7. The c/s of a beam is an I- section of overall depth 240mm,width of flanges 160mm,thickness of both flanges and web 20mm. If it carries a shear force of 70kN,draw the shear stress distribution diagram. Also find the percentage of shear carried by the web alone.

(Ans: Shear stress values at significant fibres from bottom: 0,1.69, 13.52,17.4,13.52,1.69,0. Percentage of shear carried by the web alone = 92%)

Any Questions ?

Thank you