DESIGN OF MACHINE ELEMENTES(67071)

- 7^{TH} SEMESTER
- MECHANICAL DEPARTMENT
- TECHNOLOGY CODE#70
- MYMENSINGH POLYTECHNIC INSTITUTE

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CHAPTER: 1 # Stress induced in machine elements and cause of failure

Working Stress

When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the working stress or design stress. It is also known as safe or allowable stress.

Factor of Safety

It is defined, in general, as the **ratio of the maximum stress to the working stress.** Mathematically,

$$\text{Factor of safety} = \frac{\text{Maximum stress}}{\text{Working or design stress}}$$

In case of ductile materials *e.g.* mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

$$\text{Factor of safety} = \frac{\text{Yield point stress}}{\text{Working or design stress}}$$

In case of brittle materials *e.g.* cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$\therefore \quad \text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working or design stress}}$$

This relation may also be used for ductile materials.

CHAPTER: 1 # Stress induced in machine elements and cause of failure

Selection of Factor of Safety

1. The reliability of the properties of the material and change of these properties during service ;
2. The reliability of test results and accuracy of application of these results to actual machine parts ;
3. The reliability of applied load ;
4. The certainty as to exact mode of failure ;
5. The extent of simplifying assumptions ;
6. The extent of localised stresses ;
7. The extent of initial stresses set up during manufacture ;
8. The extent of loss of life if failure occurs ; and
9. The extent of loss of property if failure occurs.

Thermal Stresses

Whenever there is some increase or decrease in the temperature of a body, it causes the body to expand or contract. A little consideration will show that if the body is allowed to expand or contract freely, with the rise or fall of the temperature, no stresses are induced in the body. But, if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are known as ***thermal stresses.***

CHAPTER: 1 # Stress induced in machine elements and cause of failure

Impact Stress

Sometimes, machine members are subjected to the load with impact. The stress produced in the member due to the falling load is known as ***impact stress.***

Consider a bar carrying a load W at a height h and falling on the collar provided at the lower end, as shown in Fig. 4.20.

Let A = Cross-sectional area of the bar,
E = Young's modulus of the material of the bar,
l = Length of the bar,
δl = Deformation of the bar,
P = Force at which the deflection δl is produced,
σi = Stress induced in the bar due to the application of impact load, and
h = Height through which the load falls

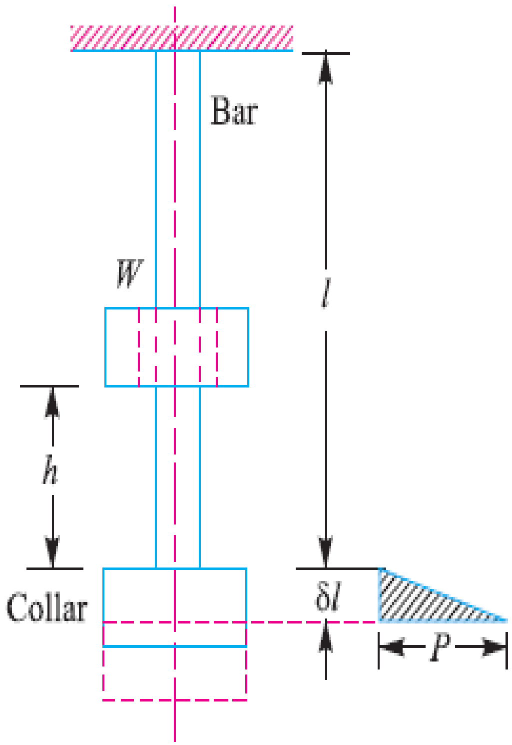


Fig. 4.20. Impact stress.

We know that energy gained by the system in the form of strain energy

$$= \frac{1}{2} \times P \times \delta l$$

and potential energy lost by the weight

$$= W(h + \delta l)$$

Since the energy gained by the system is equal to the potential energy lost by the weight, therefore

$$\frac{1}{2} \times P \times \delta l = W\,(h + \delta l)$$

$$\frac{1}{2}\sigma_i \times A \times \frac{\sigma_i \times l}{E} = W\left(h + \frac{\sigma_i \times l}{E}\right) \qquad \ldots \left[\because P = \sigma_i \times A, \text{ and } \delta l = \frac{\sigma_i \times l}{E}\right]$$

$$\therefore \qquad \frac{Al}{2E}(\sigma_i)^2 - \frac{Wl}{E}(\sigma_i) - Wh = 0$$

From this quadratic equation, we find that

$$\sigma_i = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2\,h\,A\,E}{W\,l}}\right) \qquad \ldots \text{[Taking +ve sign for maximum value]}$$

CHAPTER: 1 # Stress induced in machine elements and cause of failure

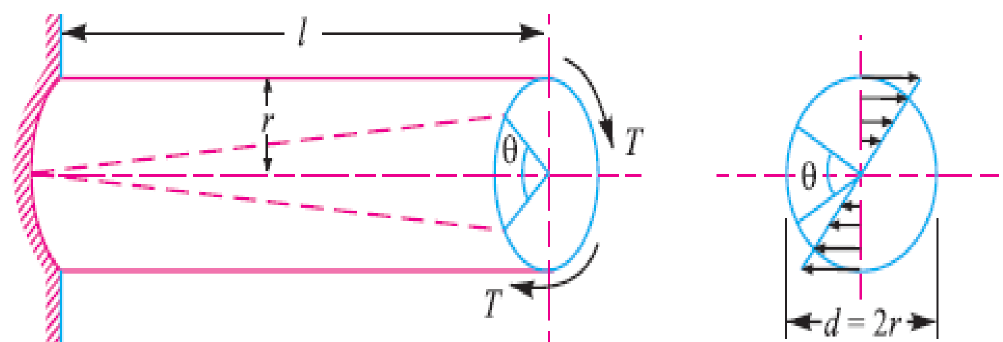
Torsional Shear Stress

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to torsion. The stress set up by torsion is known as torsional shear stress.

Consider a shaft fixed at one end and subjected to a torque (*T*) at the other end as shown in Fig. 5.1. As a result this torque, every cross-section of the shaft is subjected to torsional shear stress. The maximum torsional shear stress at the outer surface of the shaft may be obtained from the following

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l}$$

τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress,
r = Radius of the shaft,
T = Torque or twisting moment,
J = Second moment of area of the section about its polar axis or polar moment of inertia,
C = Modulus of rigidity for the shaft material,
l = Length of the shaft, and
θ = Angle of twist in radians on a length l.



Stress Concentration

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighbor hood of the discontinuity is different. This irregularity in the stress distribution caused by abrupt changes of form is called stress concentration.

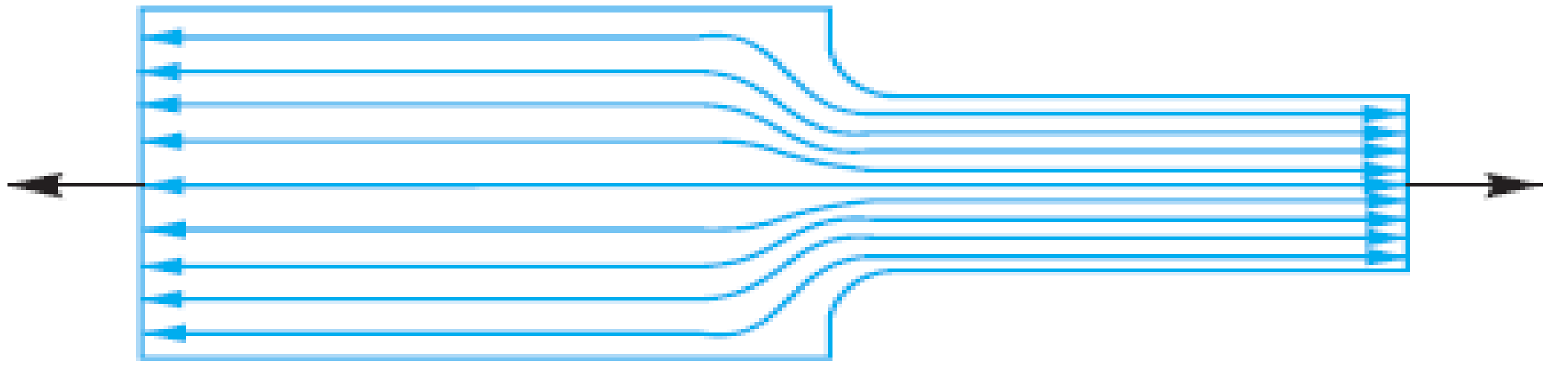


Fig. 6.5. Stress concentration.

CHAPTER: 1 # Stress induced in machine elements and cause of failure

CHAPTER: 2 Pressure Vessels

Defination: The pressure vessels (i.e. cylinders or tanks) are used to store fluids under pressure.

Classification of Pressure Vessels:

1. According to the dimensions:

The pressure vessels, according to their dimensions, may be classified as thin shell or thick shell. If the wall thickness of the shell (t) is less than 1/10 of the diameter of the shell (d), then it is called a thin shell. On the other hand, if the wall thickness

CHAPTER: 2 Pressure Vessels

2. According to the end construction.:

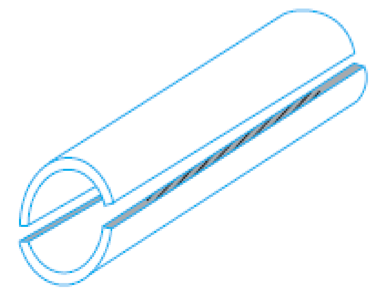
The pressure vessels, according to the end construction, may be classified as open end or closed end. A simple cylinder with a piston, such as cylinder of a press is an example of an open end vessel, whereas a tank is an example of a closed end vessel. In case of vessels having open ends, the circumferential or hoop stresses are induced by the fluid pressure, whereas in case of closed ends, longitudinal stresses in addition to circumferential stresses are induced.

Failure of a cylindrical shell:

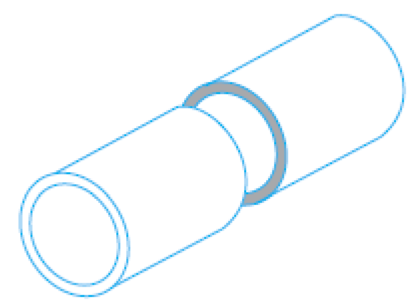
When a thin cylindrical shell is subjected to an internal pressure, it is likely to fail in the following two ways:

1. It may fail along the longitudinal section(i.e.circumferentially) splitting the cylinder into two troughs, as shown in Fig. 7.1 (a).

2. It may fail across the transverse section (i.e. longitudinally) splitting the cylinder into two cylindrical shells, as shown in Fig. 7.1 (b).



(*a*) Failure of a cylindrical shell along the longitudinal section.

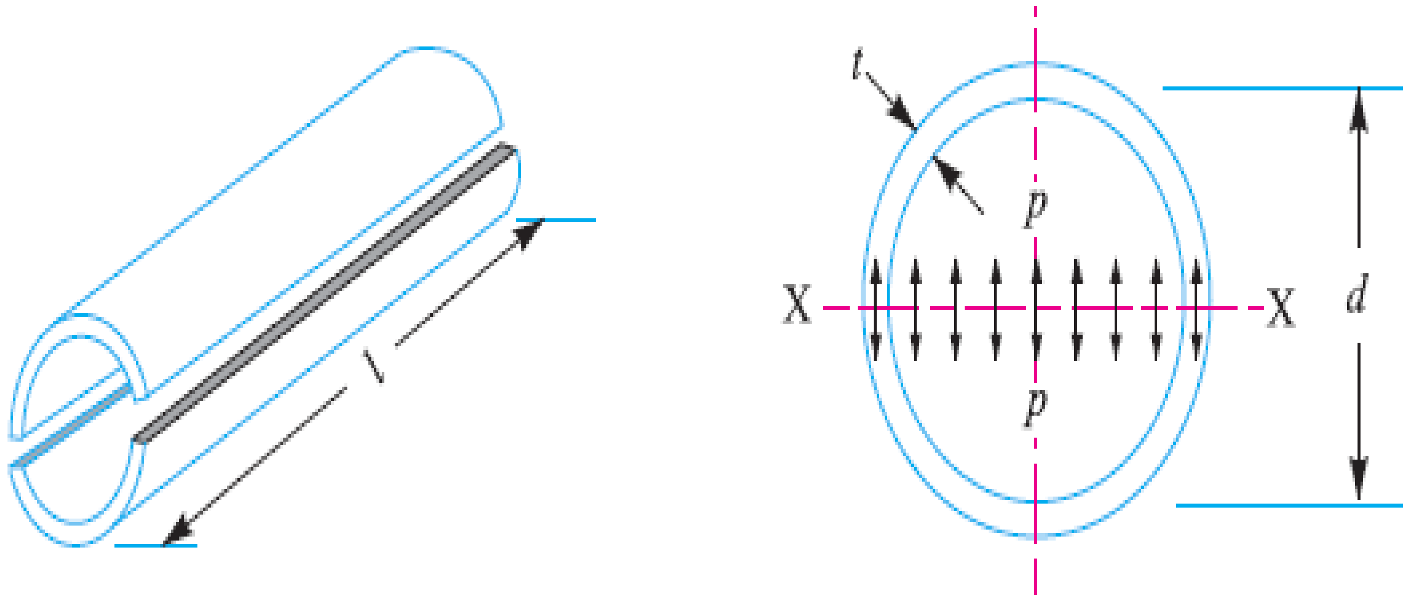


(*b*) Failure of a cylindrical shell along the transverse section.

CHAPTER: 2 Pressure Vessels

Circumferential or Hoop Stress:

Consider a thin cylindrical shell subjected to an internal pressure as shown in Fig. 7.2 (*a*) and (*b*). A tensile stress acting in a direction tangential to the circumference is called *circumferential* or *hoop stress*. In other words, it is a tensile stress on *longitudinal section (or on the cylindrical walls).



(*a*) View of shell. (*b*) Cross-section of shell.

Fig. 7.2. Circumferential or hoop stress.

CHAPTER: 2 Pressure Vessels

Let p = Intensity of internal pressure,

d = Internal diameter of the cylindrical shell,

l = Length of the cylindrical shell,

t = Thickness of the cylindrical shell, and

σ_{t1} = Circumferential or hoop stress for the material of the cylindrical shell.

We know that the total force acting on a longitudinal section (*i.e.* along the diameter *X-X*) of the shell

$$= \text{Intensity of pressure} \times \text{Projected area} = p \times d \times l \qquad \text{...}(i)$$

and the total resisting force acting on the cylinder walls

$$= \sigma_{t1} \times 2t \times l \qquad \text{...}(\because \text{ of two sections}) \qquad \text{...}(ii)$$

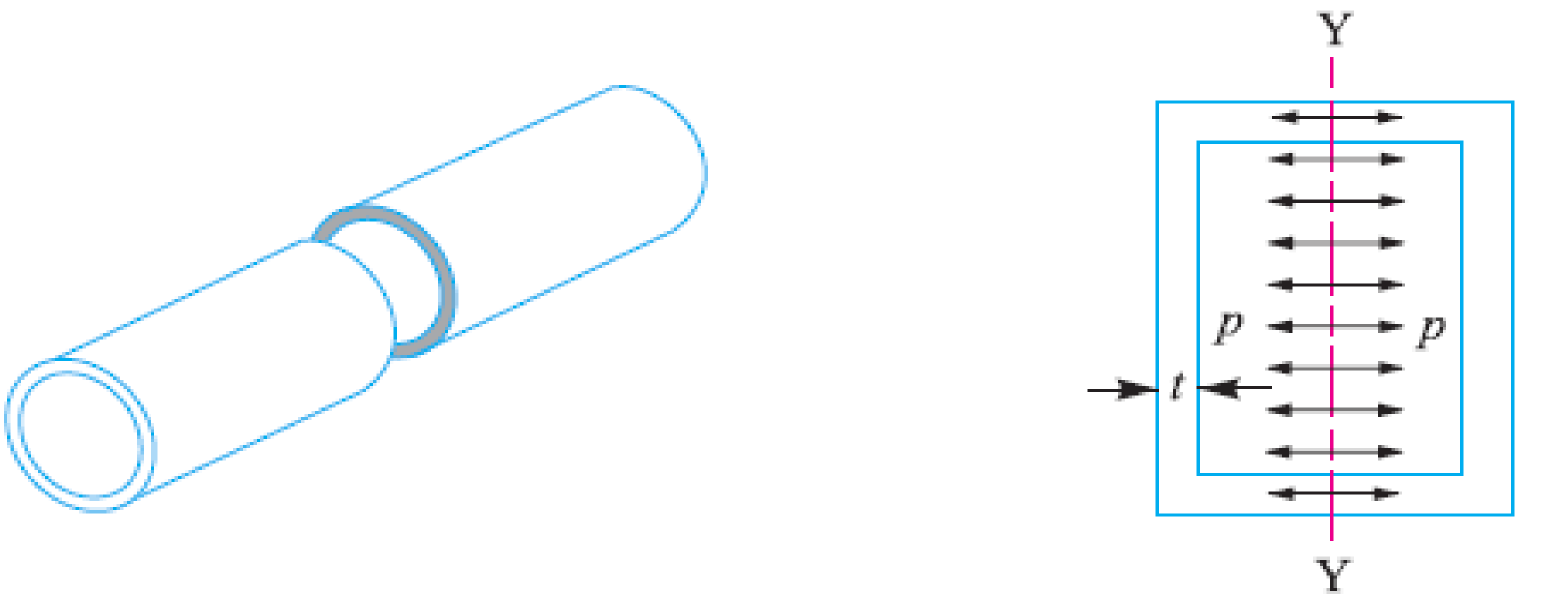
From equations (*i*) and (*ii*), we have

$$\sigma_{t1} \times 2t \times l = p \times d \times l \quad \text{or} \quad \sigma_{t1} = \frac{p \times d}{2t} \quad \text{or} \quad t = \frac{p \times d}{2\,\sigma_{t1}} \qquad \text{...}(iii)$$

CHAPTER: 2 Pressure Vessels

Longitudinal Stress

Consider a closed thin cylindrical shell subjected to an internal pressure as shown in Fig. 7.3 (a) and (b). A tensile stress acting in the direction of the axis is called longitudinal stress. In other words, it is a tensile stress acting on the *transverse or circumferential section Y-Y (or on the ends of the vessel).



(a) View of shell. (b) Cross-section of shell.

Fig. 7.3. Longitudinal stress.

CHAPTER: 2 Pressure Vessels

Let σ_{t2} = Longitudinal stress.

In this case, the total force acting on the transverse section (*i.e.* along *Y-Y*)

= Intensity of pressure × Cross-sectional area

$$= p \times \frac{\pi}{4}(d)^2 \quad \text{...}(i)$$

and total resisting force $= \sigma_{t2} \times \pi\, d.t$ ...(*ii*)

From equations (*i*) and (*ii*), we have

$$\sigma_{t2} \times \pi\, d.t = p \times \frac{\pi}{4}(d)^2$$

$$\therefore \quad \sigma_{t2} = \frac{p \times d}{4\,t} \quad \text{or} \quad t = \frac{p \times d}{4\,\sigma_{t2}}$$

If η_c is the efficiency of the circumferential joint, then

$$t = \frac{p \times d}{4\sigma_{t2} \times \eta_c}$$

From above we see that the longitudinal stress is half of the circumferential or hoop stress. Therefore, the design of a pressure vessel must be based on the maximum stress *i.e.* hoop stress.

CHAPTER: 2 Pressure Vessels

Example 7.1. *A thin cylindrical pressure vessel of 1.2 m diameter generates steam at a pressure of 1.75 N/mm2. Find the minimum wall thickness, if (a) the longitudinal stress does not*
exceed 28 MPa; and (b) the circumferential stress does not exceed 42 MPa.

Solution. Given : d = 1.2 m = 1200 mm ; p = 1.75 N/mm^2 ; σ_{t2} = 28 MPa = 28 N/mm^2 ; σ_{t1} = 42 MPa = 42 N/mm^2

(a) When longitudinal stress (σ_{t2}) does not exceed 28 MPa

We know that minimum wall thickness,

$$t = \frac{p \cdot d}{4\,\sigma_{t2}} = \frac{1.75 \times 1200}{4 \times 28} = 18.75 \text{ say } 20 \text{ mm} \quad \textbf{Ans.}$$

(b) When circumferential stress (σ_{t1}) does not exceed 42 MPa

We know that minimum wall thickness,

$$t = \frac{p \cdot d}{2\,\sigma_{t1}} = \frac{1.75 \times 1200}{2 \times 42} = 25 \text{ mm} \quad \textbf{Ans.}$$

Screw joints:

A screwed joint is mainly composed of two elements *i.e.* a bolt and nut. The screwed joints are widely used where the machine parts are required to be readily connected or disconnected without damage to the machine or the fastening. This may be for the purpose of holding or adjustment in assembly or service inspection, repair, or replacement or it may be for the manufacturing or assembly reasons.

Advantages and Disadvantages of Screwed Joints

Advantages

1. Screwed joints are highly reliable in operation.
2. Screwed joints are convenient to assemble and disassemble.
3. A wide range of screwed joints may be adopted to various operating conditions.
4. Screws are relatively cheap to produce due to standardisation and highly efficient manufacturing processes.

Disadvantages

The main disadvantage of the screwed joints is the stress concentration in the threaded portions which are vulnerable points under variable load conditions.

Important Terms Used in Screw Threads

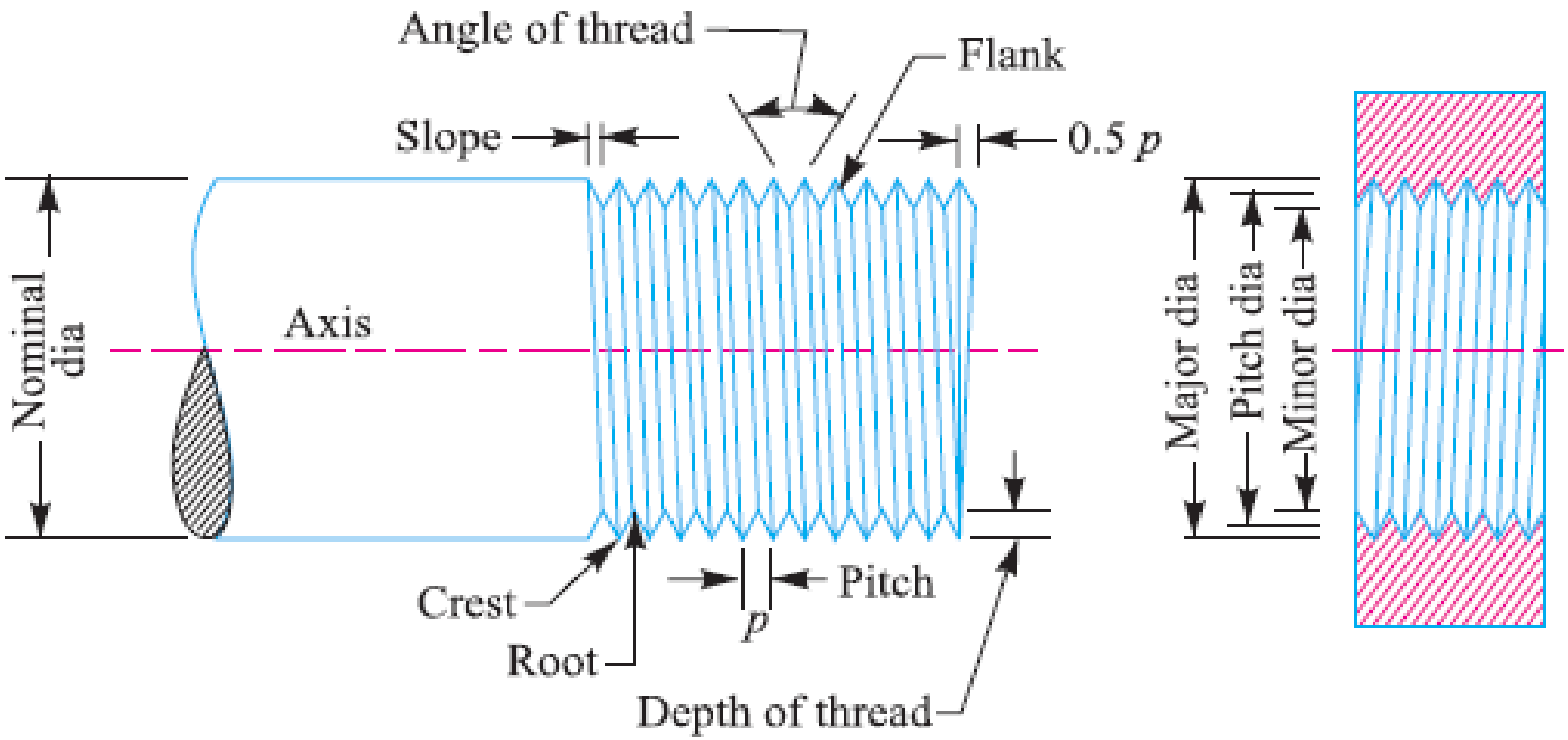
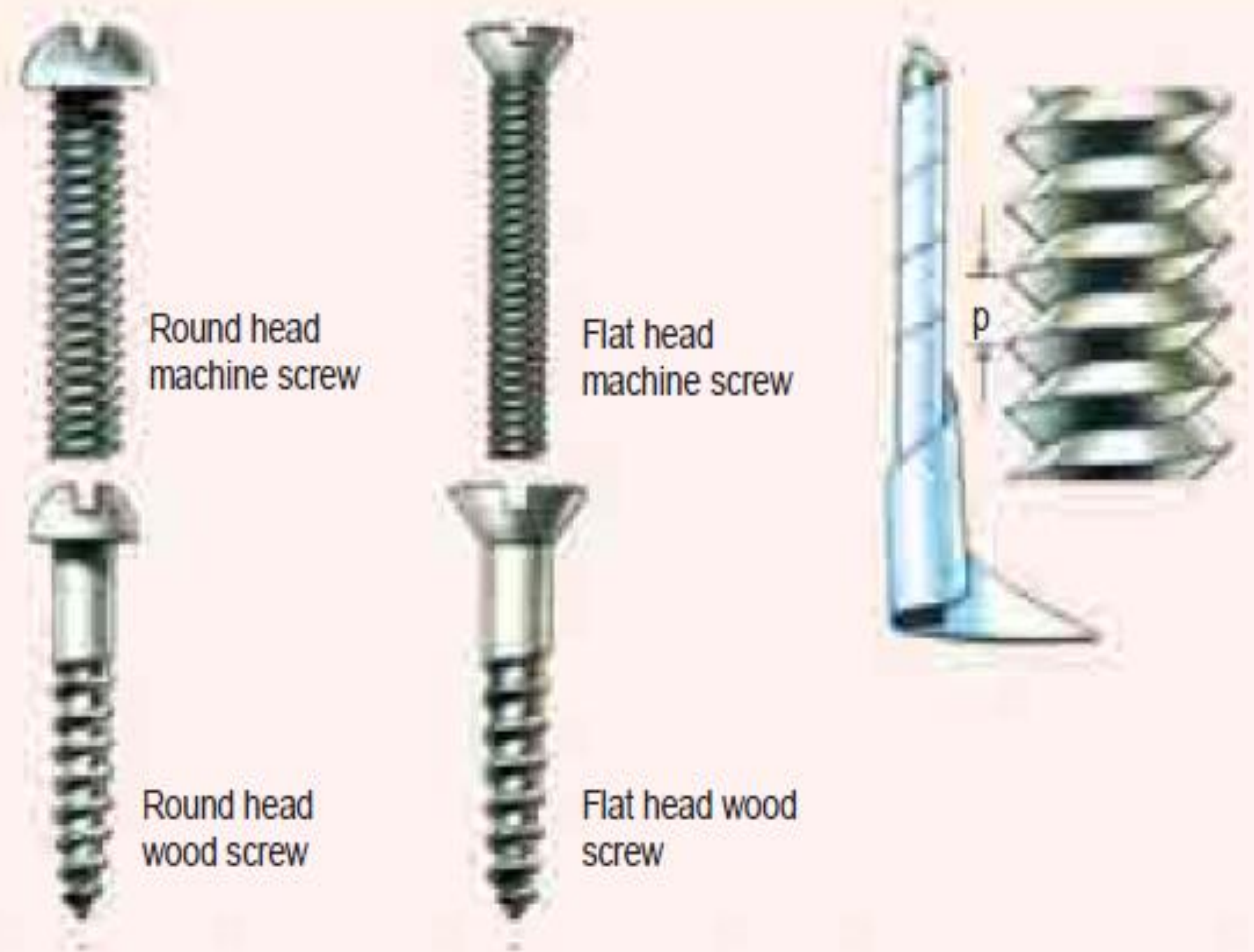


Fig. 11.1. Terms used in screw threads.


Round head
machine screw
Flat head
machine screw
p
Round head
wood screw
Flat head wood
screw

Stresses in Screwed Fastening due to Static Loading

The following stresses in screwed fastening due to static loading are important from the subject
point of view :

1. Internal stresses due to screwing up forces,

2. Stresses due to external forces, and

3. Stress due to combination of stresses at (1) and (2).

Eccentric Load Acting Parallel to the Axis of Bolts

Consider a bracket having a rectangular base bolted to a wall by means of four bolts as shown in Fig. . A little consideration will show that each bolt is subjected to a direct tensile load of
$W_{t1} = W/n$, where *n* is the number of bolts.
Let w be the load in a bolt per unit distance due to the turning effect of the bracket and let W1 and W2 be the loads on each of the bolts at distances L1 and L2 from the tilting edge.

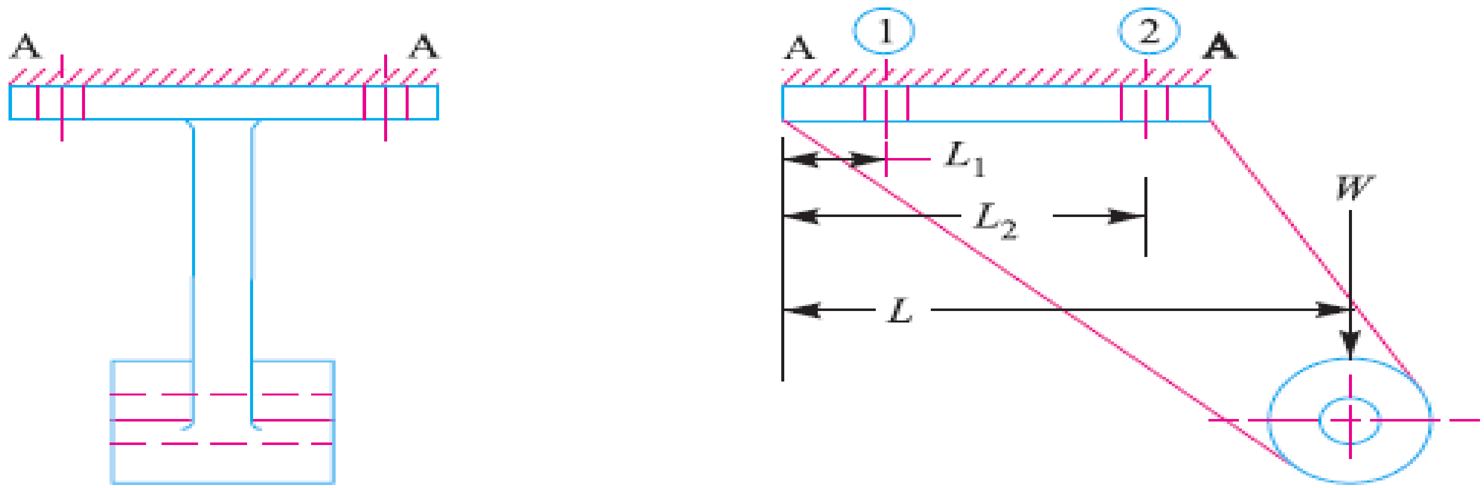


Fig. 11.31. Eccentric load acting parallel to the axis of bolts.

∴ Load on each bolt at distance L1,
W1 = w.L1
and moment of this load about the tilting edge
= w1.L1 × L1 = w (L1)2
Similarly, load on each bolt at distance L2,
W2 = w.L2
and moment of this load about the tilting edge
= w.L2 × L2 = w (L2)2
∴ Total moment of the load on the bolts about the tilting edge
= 2w (L1)2 + 2w (L2)2 ...(i)

Also the moment due to load W about the tilting edge

$$= W.L \qquad \text{...}(ii)$$

From equations (*i*) and (*ii*), we have

$$W.L = 2w\,(L_1)^2 + 2w(L_2)^2 \quad \text{or} \quad w = \frac{W.L}{2\,[(L_1)^2 + (L_2)^2]} \qquad \text{...}(iii)$$

It may be noted that the most heavily loaded bolts are those which are situated at the greatest distance from the tilting edge. In the case discussed above, the bolts at distance L_2 are heavily loaded.

∴ Tensile load on each bolt at distance L_2,

$$W_{t2} = W_2 = w.L_2 = \frac{W.L.L_2}{2[(L_1)^2 + (L_2)^2]} \qquad \text{... [From equation } (iii)]$$

and the total tensile load on the most heavily loaded bolt,

$$W_t = W_{t1} + W_{t2} \qquad \text{...}(iv)$$

If d_c is the core diameter of the bolt and σ_t is the tensile stress for the bolt material, then total tensile load,

$$W_t = \frac{\pi}{4}\,(d_c)^2\,\sigma_t \qquad \text{...}(v)$$

From equations (*iv*) and (*v*), the value of d_c may be obtained.

Eccentric Load Acting Perpendicular to the Axis of Bolts

A wall bracket carrying an eccentric load perpendicular to the axis of the bolts is shown in Fig. 11.34.

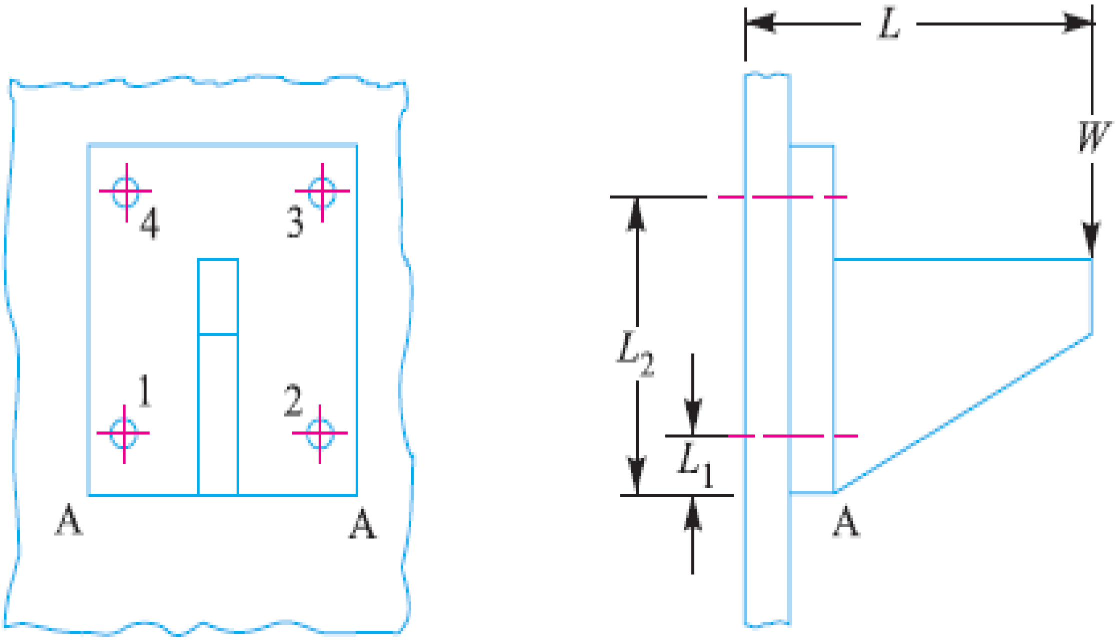


Fig. 11.34. Eccentric load perpendicular to the axis of bolts.

In this case, the bolts are subjected to direct shearing load which is equally shared by all the bolts. Therefore direct shear load on each bolts,

$W_s = W/n$, where n is number of bolts.

$\therefore$ Maximum tensile load on bolt 3 or 4,

$$W_{t2} = W_t = \frac{W.L.L_2}{2\,[(L_1)^2 + (L_2)^2]}$$

When the bolts are subjected to shear as well as tensile loads, then the equivalent loads may be determined by the following relations :

Equivalent tensile load,

$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right]$$

and equivalent shear load,

$$W_{se} = \frac{1}{2}\left[\sqrt{(W_t)^2 + 4(W_s)^2}\right]$$

Knuckle Joint

A knuckle joint is used to connect two rods which are under the action of tensile loads. However, if the joint is guided, the rods may support a compressive load. A knuckle joint may be readily disconnected for adjustments or repairs. Its use may be found in the link of a cycle chain, tie rod joint for roof truss, valve rod joint with eccentric rod, pump rod joint, tension link in bridge structure and lever and rod connections of various types.

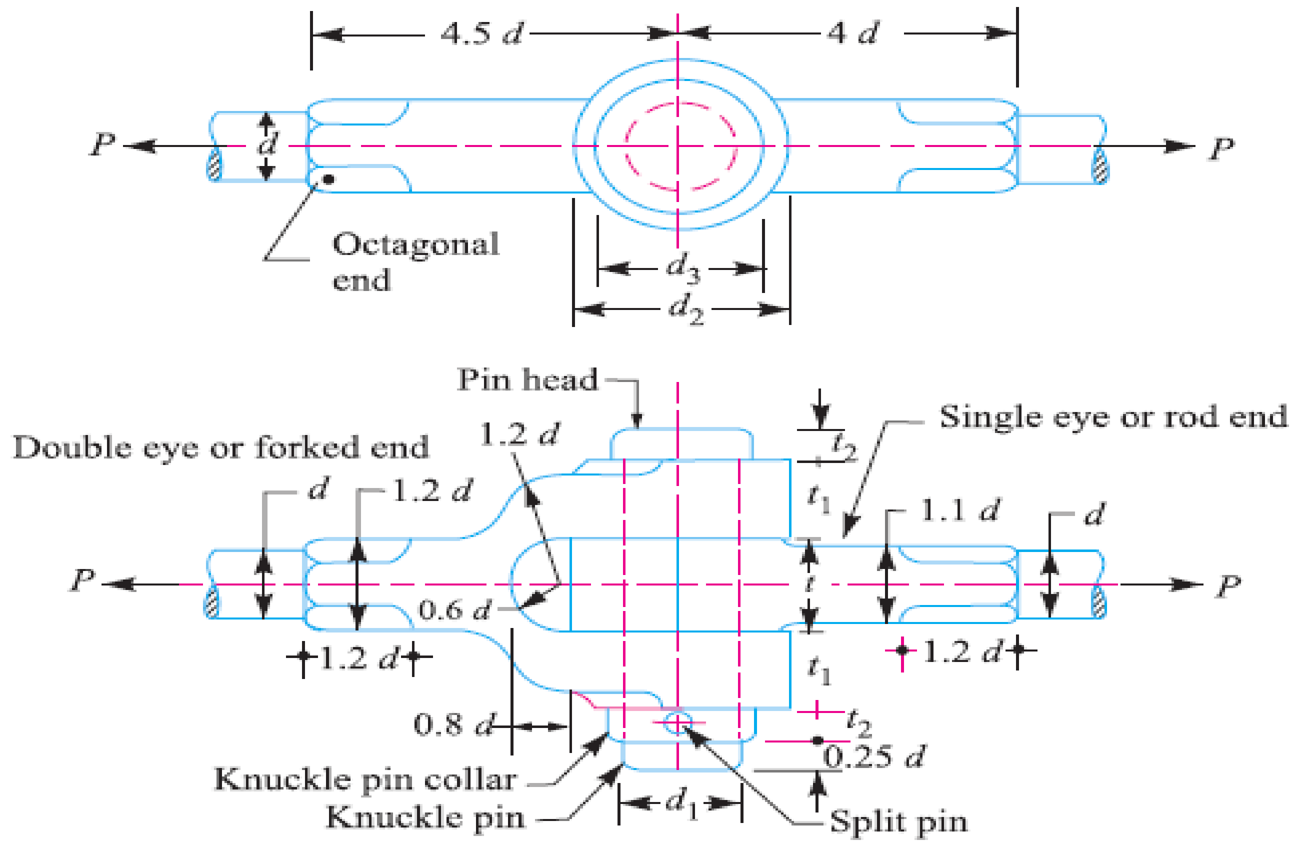


Fig. 12.16. Kunckle joint.

Dimensions of Various Parts of the Knuckle Joint:

If d is the diameter of rod, then diameter of pin, $d_1 = d$

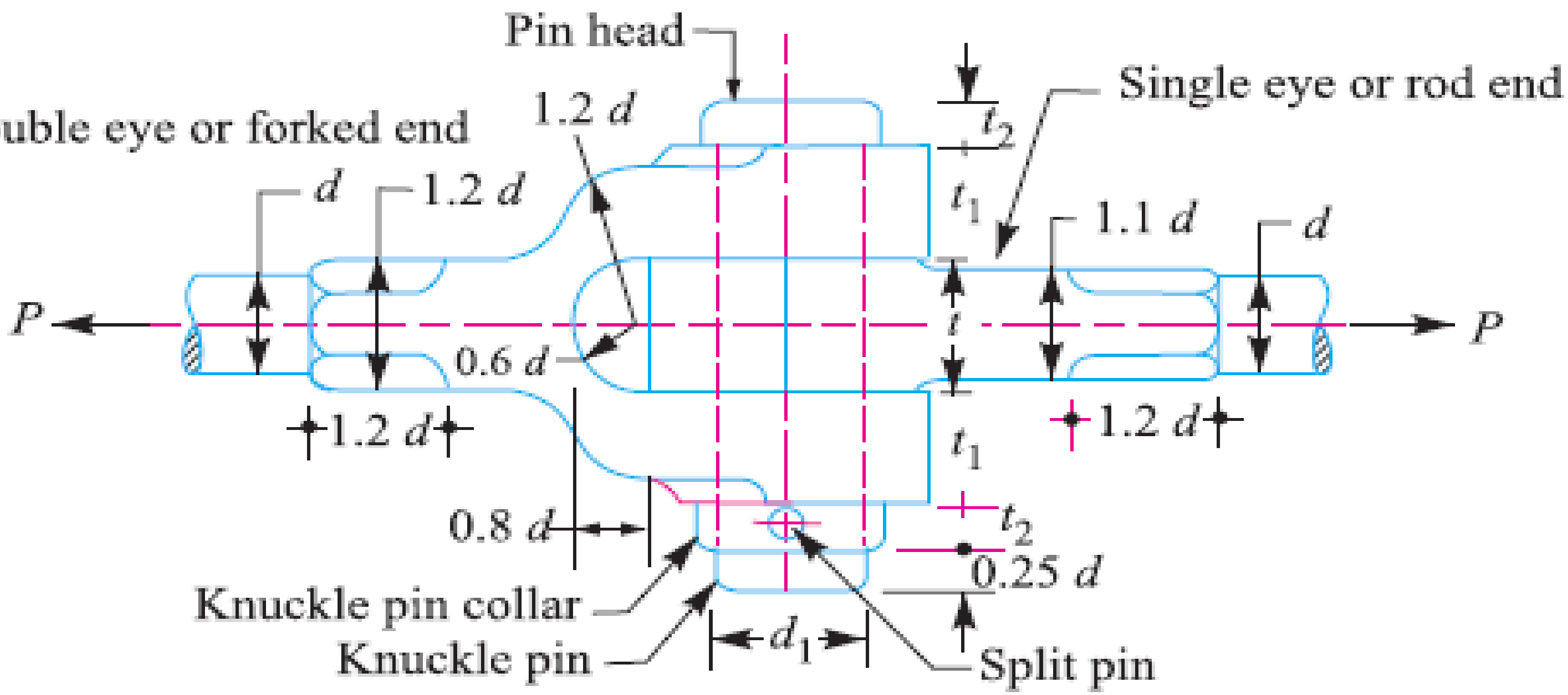
Outer diameter of eye, $d_2 = 2\,d$

Diameter of knuckle pin head and collar, $d_3 = 1.5\,d$

Thickness of single eye or rod end, $t = 1.25\,d$

Thickness of fork, $t_1 = 0.75\,d$

Thickness of pin head, $t_2 = 0.5\,d$



Methods of Failure of Knuckle Joint:

1. *Failure of the solid rod in tension*

Since the rods are subjected to direct tensile load, therefore tensile strength of the rod,

$$= \frac{\pi}{4} \times d^2 \times \sigma_t$$

Equating this to the load (P) acting on the rod, we have

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rod (d) is obtained.

2. *Failure of the knuckle pin in shear*

Since the pin is in double shear, therefore cross-sectional area of the pin under shearing

$$= 2 \times \frac{\pi}{4} (d_1)^2$$

and the shear strength of the pin

$$= 2 \times \frac{\pi}{4} (d_1)^2 \ \tau$$

Equating this to the load (P) acting on the rod, we have

$$P = 2 \times \frac{\pi}{4} (d_1)^2 \ \tau$$

In case, the stress due to bending is taken into account, it is assumed that the load on the pin is uniformly distributed along the middle portion (*i.e.* the eye end) and varies uniformly over the forks as shown in Fig. 12.17. Thus in the forks, a load $P/2$ acts through a distance of $t_1 / 3$ from the inner edge and the bending moment will be maximum at the centre of the pin. The value of maximum bending moment is given by

$$M = \frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{2}\right) - \frac{P}{2} \times \frac{t}{4}$$

$$= \frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{2} - \frac{t}{4}\right)$$

$$= \frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{4}\right)$$

and section modulus, $Z = \frac{\pi}{32}(d_1)^3$

$\therefore$ Maximum bending (tensile) stress,

$$\sigma_t = \frac{M}{Z} = \frac{\frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{4}\right)}{\frac{\pi}{32}(d_1)^3}$$

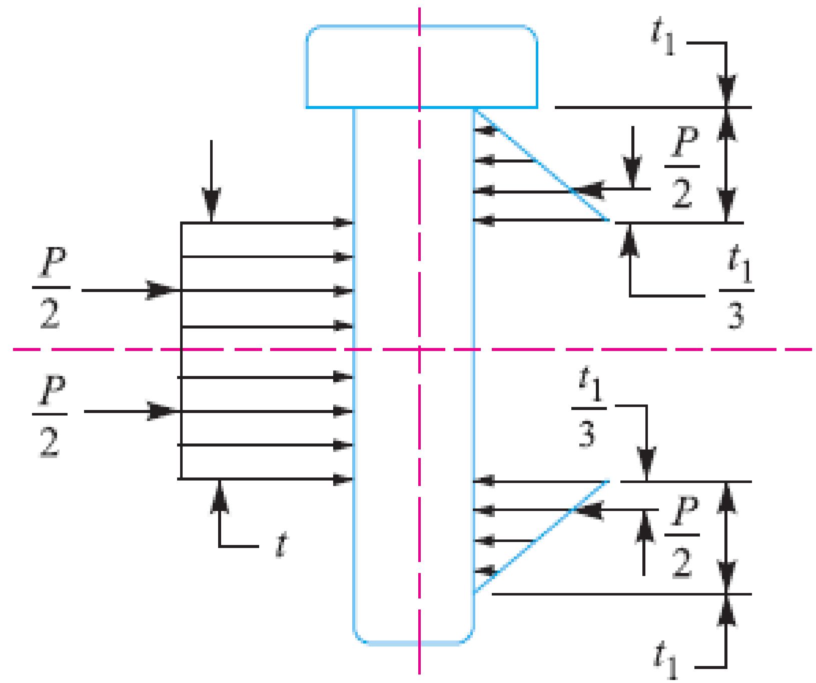


Fig. 12.17. Distribution of load on the pin.

From this expression, the value of d_1 may be obtained.

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$$M = \frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{2}\right) - \frac{P}{2} \times \frac{t}{4}$$

$$= \frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{2} - \frac{t}{4}\right)$$

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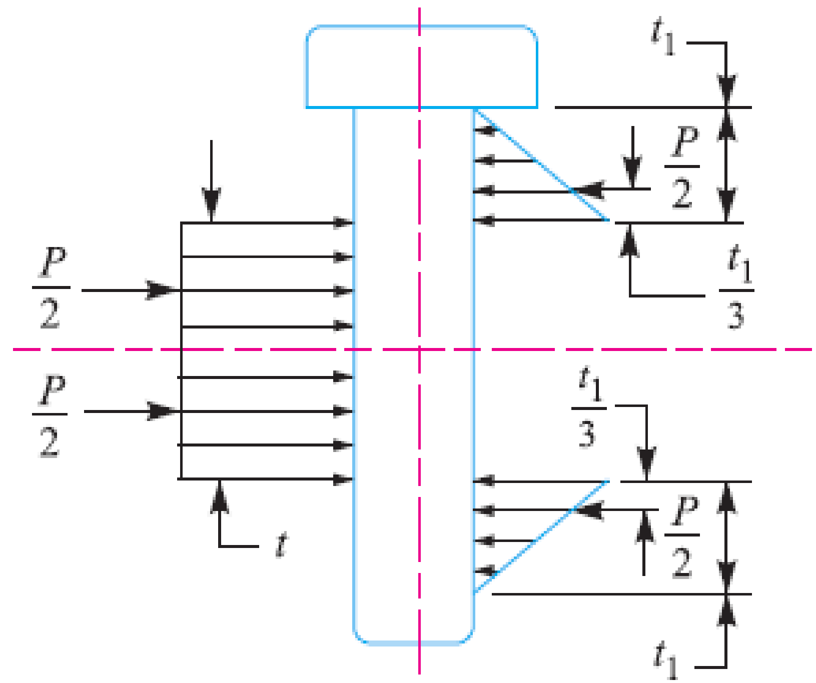


Fig. 12.17. Distribution of load on the pin.

From this expression, the value of d_1 may be obtained.

3. *Failure of the single eye or rod end in tension*

The single eye or rod end may tear off due to the tensile load. We know that area resisting tearing $= (d_2 - d_1)\, t$

$\therefore$ Tearing strength of single eye or rod end

$$= (d_2 - d_1)\, t \times \sigma_t$$

Equating this to the load (P) we have

$$P = (d_2 - d_1)\, t \times \sigma_t$$

From this equation, the induced tensile stress (σ_t) for the single eye or rod end may be checked. In case the induced tensile stress is more than the allowable working stress, then increase the outer diameter of the eye (d_2).

4. *Failure of the single eye or rod end in shearing*

The single eye or rod end may fail in shearing due to tensile load. We know that area resisting shearing $= (d_2 - d_1)\, t$

$\therefore$ Shearing strength of single eye or rod end

$$= (d_2 - d_1)\, t \times \tau$$

Equating this to the load (P), we have

$$P = (d_2 - d_1)\, t \times \tau$$

From this equation, the induced shear stress (τ) for the single eye or rod end may be checked.

5. Failure of the single eye or rod end in crushing

The single eye or pin may fail in crushing due to the tensile load. We know that area resisting crushing $= d_1 \times t$

$\therefore$ Crushing strength of single eye or rod end

$$= d_1 \times t \times \sigma_c$$

Equating this to the load (P), we have

$$\therefore \quad P = d_1 \times t \times \sigma_c$$

6. Failure of the forked end in tension

The forked end or double eye may fail in tension due to the tensile load. We know that area resisting tearing

$$= (d_2 - d_1) \times 2\, t_1$$

$\therefore$ Tearing strength of the forked end

$$= (d_2 - d_1) \times 2\, t_1 \times \sigma_t$$

Equating this to the load (P), we have

$$P = (d_2 - d_1) \times 2t_1 \times \sigma_t$$

From this equation, the induced tensile stress for the forked end may be checked.

7. Failure of the forked end in shear

The forked end may fail in shearing due to the tensile load. We know that area resisting shearing

$$= (d_2 - d_1) \times 2t_1$$

$\therefore$ Shearing strength of the forked end

$$= (d_2 - d_1) \times 2t_1 \times \tau$$

Equating this to the load (P), we have

$$P = (d_2 - d_1) \times 2t_1 \times \tau$$

8. *Failure of the forked end in crushing*

The forked end or pin may fail in crushing due to the tensile load. We know that area resisting crushing $= d_1 \times 2\, t_1$

$\therefore$ Crushing strength of the forked end

$$= d_1 \times 2\, t_1 \times \sigma_c$$

Equating this to the load (P), we have

$$P = d_1 \times 2\, t_1 \times \sigma_c$$

Example 12.7. *Design a knuckle joint to transmit 150 kN. The design stresses may be taken as 75 MPa in tension, 60 MPa in shear and 150 MPa in compression.*

Solution. Given : $P = 150\text{ kN} = 150 \times 10^3\text{ N}$; $\sigma_t = 75\text{ MPa} = 75\text{ N/mm}^2$; $\tau = 60\text{ MPa} = 60\text{ N/mm}^2$; $\sigma_c = 150\text{ MPa} = 150\text{ N/mm}^2$

The knuckle joint is shown in Fig. 12.16. The joint is designed by considering the various methods of failure as discussed below :

CHAPTER: 4 knuckle joint

1. *Failure of the solid rod in tension*

Let d = Diameter of the rod.

We know that the load transmitted (P),

$$150 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 75 = 59\, d^2$$

$$\therefore \quad d^2 = 150 \times 10^3 / 59 = 2540 \quad \text{or} \quad d = 50.4 \text{ say } 52 \text{ mm} \quad \textbf{Ans.}$$

Now the various dimensions are fixed as follows :

Diameter of knuckle pin,

$$d_1 = d = 52 \text{ mm}$$

Outer diameter of eye, $d_2 = 2\,d = 2 \times 52 = 104$ mm

Diameter of knuckle pin head and collar,

$$d_3 = 1.5\, d = 1.5 \times 52 = 78 \text{ mm}$$

Thickness of single eye or rod end,

$$t = 1.25\, d = 1.25 \times 52 = 65 \text{ mm}$$

Thickness of fork, $t_1 = 0.75\, d = 0.75 \times 52 = 39$ say 40 mm

Thickness of pin head, $t_2 = 0.5\, d = 0.5 \times 52 = 26$ mm

2. *Failure of the knuckle pin in shear*

Since the knuckle pin is in double shear, therefore load (P),

$$150 \times 10^3 = 2 \times \frac{\pi}{4} \times (d_1)^2\, \tau = 2 \times \frac{\pi}{4} \times (52)^2\, \tau = 4248\, \tau$$

$$\therefore \quad \tau = 150 \times 10^3 / 4248 = 35.3 \text{ N/mm}^2 = 35.3 \text{ MPa}$$

3. Failure of the single eye or rod end in tension

The single eye or rod end may fail in tension due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, t \times \sigma_t = (104 - 52)\, 65 \times \sigma_t = 3380\, \sigma_t$$

$$\therefore \quad \sigma_t = 150 \times 10^3 / 3380 = 44.4 \text{ N / mm}^2 = 44.4 \text{ MPa}$$

4. Failure of the single eye or rod end in shearing

The single eye or rod end may fail in shearing due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, t \times \tau = (104 - 52)\, 65 \times \tau = 3380\, \tau$$

$$\therefore \quad \tau = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4 \text{ MPa}$$

5. Failure of the single eye or rod end in crushing

The single eye or rod end may fail in crushing due to the load. We know that load (P),

$$150 \times 10^3 = d_1 \times t \times \sigma_c = 52 \times 65 \times \sigma_c = 3380\, \sigma_c$$

$$\therefore \quad \sigma_c = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4 \text{ MPa}$$

6. Failure of the forked end in tension

The forked end may fail in tension due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, 2\, t_1 \times \sigma_t = (104 - 52)\, 2 \times 40 \times \sigma_t = 4160\, \sigma_t$$

$$\therefore \quad \sigma_t = 150 \times 10^3 / 4160 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

7. Failure of the forked end in shear

The forked end may fail in shearing due to the load. We know that load (P),

$$150 \times 10^3 = (d_2 - d_1)\, 2\, t_1 \times \tau = (104 - 52)\, 2 \times 40 \times \tau = 4160\, \tau$$

$$\therefore \quad \tau = 150 \times 10^3 / 4160 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

8. Failure of the forked end in crushing

The forked end may fail in crushing due to the load. We know that load (P),

$$150 \times 10^3 = d_1 \times 2\, t_1 \times \sigma_c = 52 \times 2 \times 40 \times \sigma_c = 4160\, \sigma_c$$

$$\therefore \quad \sigma_c = 150 \times 10^3 / 4180 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

Shaft:

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft.

Axle :

An axle, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

Spindle

A spindle is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (e.g. lathe spindles).

Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are :
25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps ; 110 mm to 140 mm with 15 mm steps ; and 140 mm to 500 mm with 20 mm steps.
The standard length of the shafts are 5 m, 6 m and 7 m.

Stresses in Shafts

The following stresses are induced in the shafts :

1. Shear stresses due to the transmission of torque (i.e. due to torsional load).
2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
3. Stresses due to combined torsional and bending loads

Design of Shafts

The shafts may be designed on the basis of
1. Strength, and **2.** Rigidity and stiffness.

Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2} = \frac{16}{\pi d^3}\left[\sqrt{M^2 + T^2}\right]$$

or $$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \qquad \text{...}(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (*i*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{...}(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad \text{...}(iii)$$

$$= \frac{1}{2} \times \frac{32M}{\pi d^3} + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$= \frac{32}{\pi d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right]$$

or $$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad \text{...}(iv)$$

The expression $\frac{1}{2}\left[(M + \sqrt{M^2 + T^2})\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \qquad \ldots(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations (*ii*) and (*v*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

and

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1 - k^4)$$

Keys

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them.

A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key.

Types of Keys

The following types of keys are important from the subject point of view :
1. Sunk keys, 2. Saddle keys, 3. Tangent keys, 4. Round keys, and 5. Splines.

Belt Drives

- Belt drives are called flexible machine elements.
- Used in conveying systems
- Used for transmission of power.
- Replacement of rigid type power transmission system.

Open Belt Drive

13.1.3 Nomenclature of Open Belt Drive

d_L - Diameter of the larger pulley

d_S – Diameter of the smaller pulley

α_L- Angle of wrap of the larger pulley

α_S – Angle of wrap of the smaller pulley

C- Center distance between the two pulleys

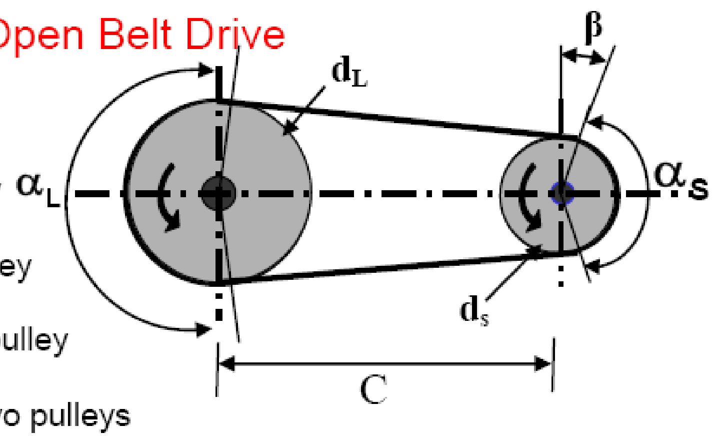


Fig.13.1.1 Open belt drive

Open Belt Drive

Basic Formulae

$\alpha_L = 180^o + 2\beta$

$\alpha_S = 180^o - 2\beta$

Where angle β is,

$$\beta = \sin^{-1}\left(\frac{d_L - d_S}{2C}\right)$$

L_0 = Length of open belt

$$L_o = \frac{\pi}{2}(d_L + d_S) + 2C + \frac{1}{4C}(d_L - d_S)^2$$

Cross Belt drive

13.1.4 Nomenclature of Cross Belt Drive

d_L - Diameter of the larger pulley

d_S – Diameter of the smaller pulley

α_L- Angle of wrap of the larger pulley

α_S – Angle of wrap of the smaller pulley

C- Center distance between the two pulleys

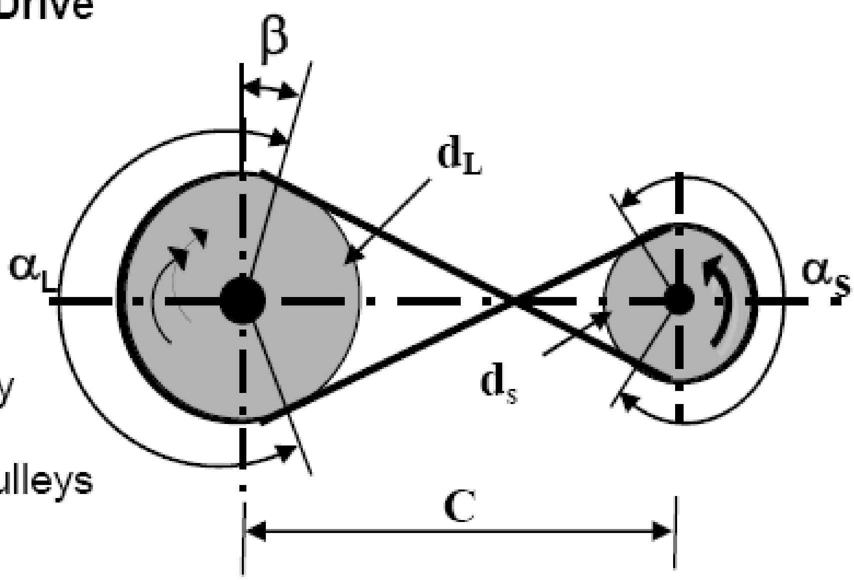


Fig. 13.1.2 Cross belt drive

Cross Belt Drive

Basic Formulae

$\alpha_L = \alpha_S = 180^\circ + 2\beta$

Where angle β is,

$$\beta = \sin^{-1}\left(\frac{d_L - d_S}{2C}\right)$$

Length of cross belt

$$L_c = \frac{\pi}{2}\left(d_L + d_S\right) + 2C + \frac{1}{4C}\left(d_L + d_S\right)^2$$

Relationship between belt tensions

- The equation for determination of rela

$$\frac{T_1 - mv^2}{T_2 - mv^2} = e^{\mu\alpha}$$

Belt types

- Flat belts *made from joined hides were first on the scene, however modern flat belts are of composite construction with cord reinforcement. They are particularly suitable for high speeds.*

Belt types

- Classical banded (ie. covered) *V-belts* comprise cord tensile members located at the pitchline, embedded in a relatively soft matrix which is encased in a wear resistant cover. The wedging action of a V-belt in a pulley groove results in a drive which is more compact than a flat belt drive, but short centre V-belt drives are not conducive to shock absorption

Belt types

- *Wedge belts* are narrower and thus lighter than V-belts. Centrifugal effects which reduce belt-pulley contact pressure and hence frictional torque are therefore not so deleterious in wedge belt drives as they are in V-belt drives.

Belt types

- Modern materials allow *cut* belts to dispense with a separate cover. The belt illustrated also incorporates slots on the underside known as ***cogging*** which alleviate deleterious bending stresses as the belt is forced to conform to pulley curvature.

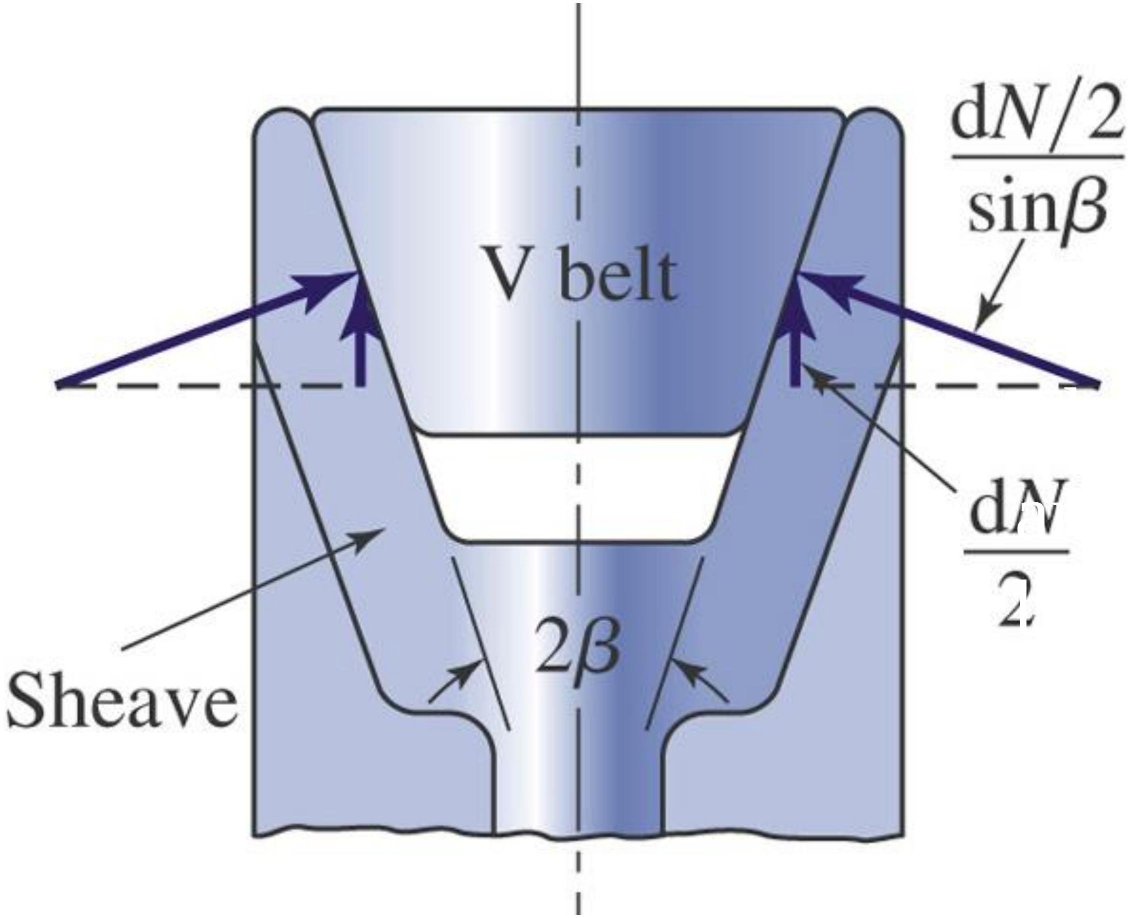
Belt types

- *Synchronous* or *timing belt* drives are positive rather than friction drives as they rely on gear- like teeth on pulley and belt enabled by modern materials and manufacturing methods

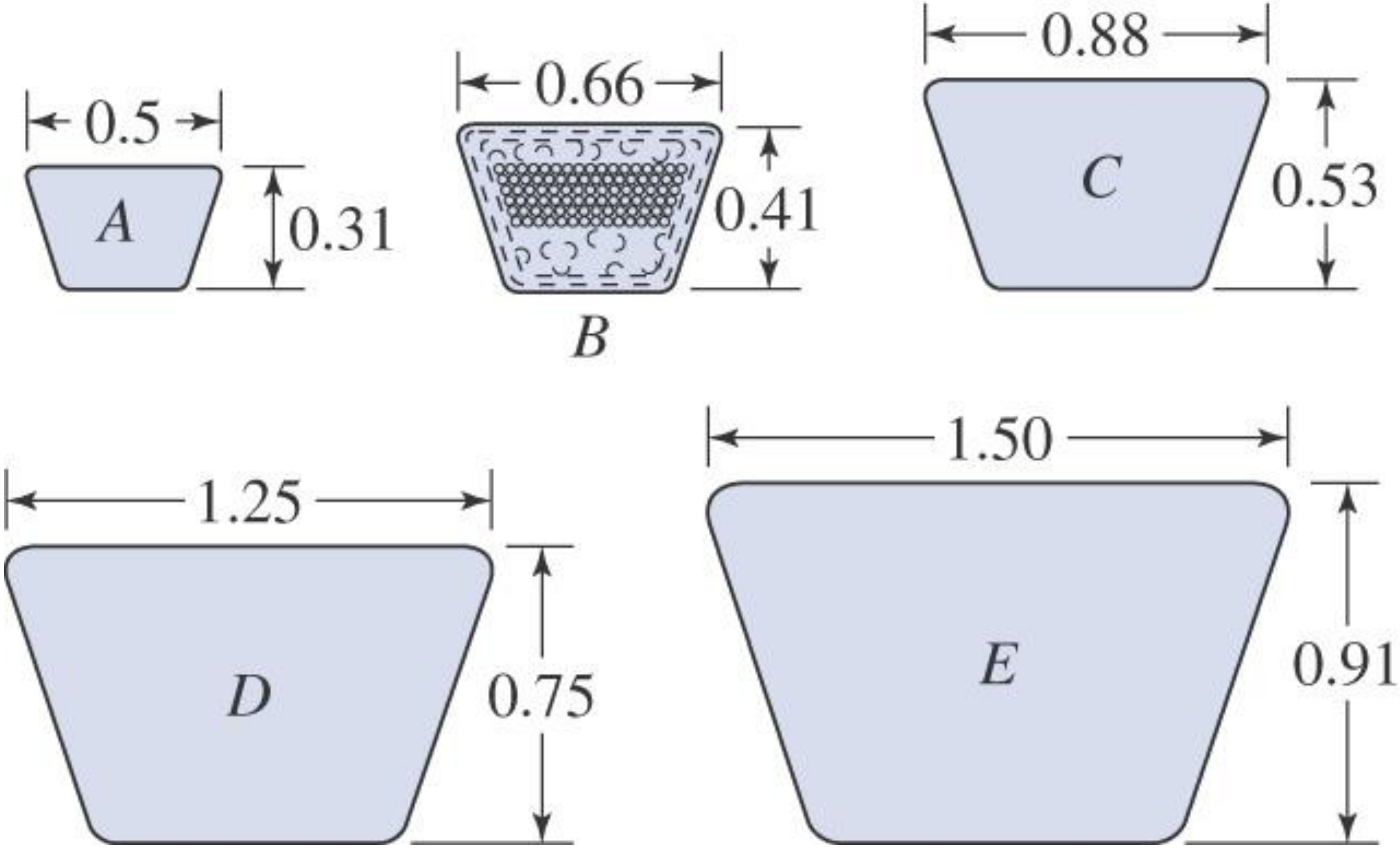
Flat, Round, and V Belts

- Flat and round belts work very well. Flat belts must work under higher tension than V belts to transmit the same torque as V belts. Therefore they require more rigid shafts, larger bearings, and so on.
- V belts create greater friction by wedging into the groove on the pulley or *sheave*. This greater friction = great torque capacity.

V Belt & Sheave Cross Section



V Belt Cross Sections (U.S.)



Flat Belts vs. V Belts

- Flat belt drives can have an efficiency close to 98%, about the same as a gear drive.
- V belt drive efficiency varies between 70% and 96%, but they can transmit more power for a similar size. (Think of the wedged belt having to come un-wedged.)
- In low power applications (most industrial uses), the cheaper installed cost wins vs. their greater efficiency: V belts are very common.

Flat Belt Drives

- Flat belts drives can be used for large amount of power transmission and there is no upper limit of distance between the two pulleys.
- Idler pulleys are used to guide a flat belt in various manners, but do not contribute to power transmission
- These drives are efficient at high speeds and they offer quite running.

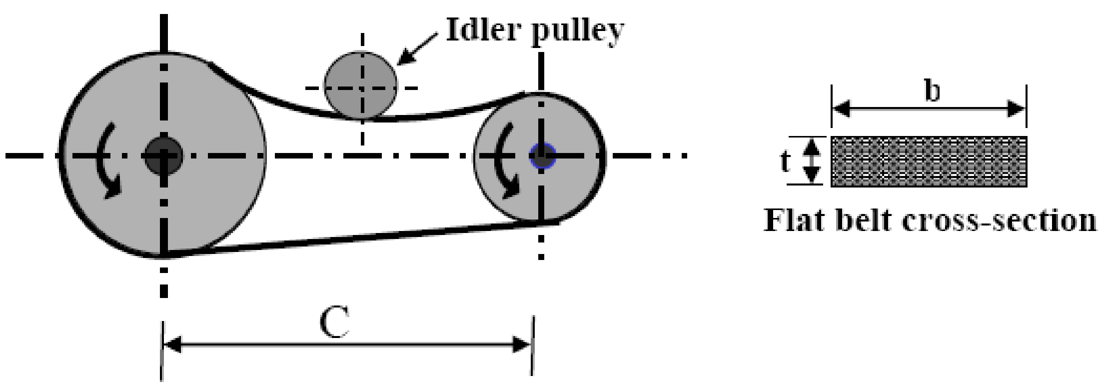


Fig.13.2.1 Belt drive with idler

Belt Materials

- The belt material is chosen depending on the use and application. Leather oak tanned belts and rubber belts are the most commonly used but the plastic belts have a very good strength almost twice the strength of leather belt. Fabric belts are used for temporary or short period operations.

Typical Belt Drive Specifications

Belts are specified on the following parameters

- Material
- No of ply and thickness
- Maximum belt stress per unit width
- Density of the belt material
- Coefficient of friction of the belt material

Design Factors

- A belt drive is designed based on the design power, which is the modified required power. The modification factor is called the service factor. The service factor depends on hours of running, type of shock load expected and nature of duty.

Hence**,**

Design Power (P_d) = service factor (C_s)* Required Power (P)

C_s = 1.1 to 1.8 for light to heavy shock.

Design Factors

- Speed correction factor(C_S)- When Maximum belt stress/ unit width is given for a specified speed, a speed correction factor ($\mathbf{C_S}$) is required to modify the belt stress when the drive is operating at a speed other than the specified one.
- Angle of wrap correction factor(C_W)-The maximum stress values are given for an angle of wrap is 180^o for both the pulleys, ie, pulleys are of same diameter. Reduction of belt stress is to be considered for angle of wrap less than 180^o.

Design Factors

From the $\mathbf{bt}\left(\sigma' - \rho v^2\right)\left(1 - \frac{1}{e^{\mu\alpha}}\right)v$ e, it can be shown that,

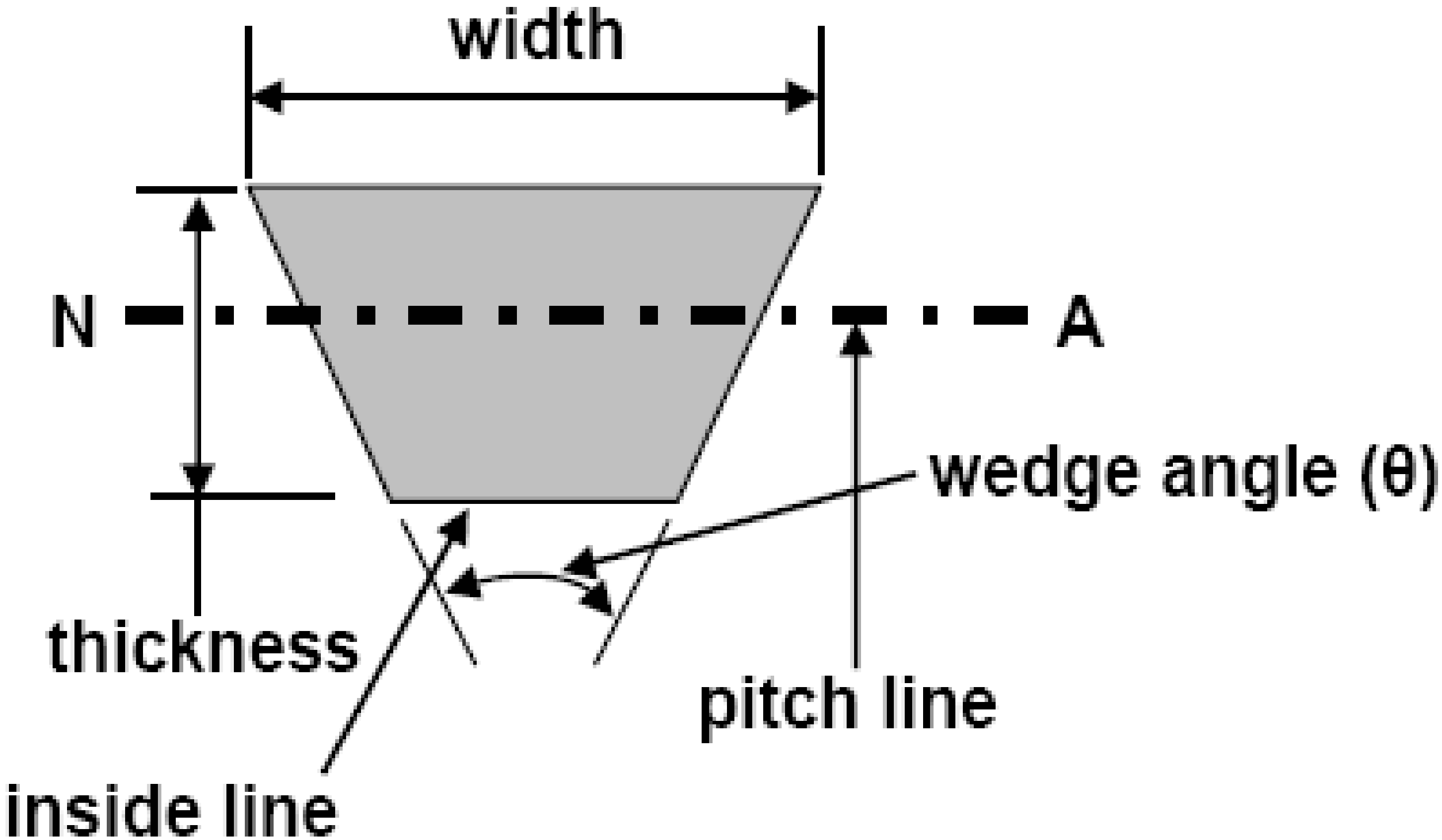
P_d =

where $\sigma' = \sigma_{max}\, C_S\, C_w$

Selection of Flat Belt

- Transmission ratio of flat belt drives is normally limited to 1:5
- Centre distance is dependent on the available space. In the case of flat belt drives there is not much limitation of centre distance. Generally the centre distance is taken as more than twice the sum of the pulley diameters.
- Depending on the driving and driven shaft speeds, pulley diameters are to be calculated and selected from available standard sizes.
- Belt speed is recommended to be within 15-25 m/s.
- Finally, the calculated belt length is normally kept 1% short to account for correct initial tension.

Nomenclature of V-Belts



Standard V-Belt Sections

- The standard V-belt sections are A, B, C, D and E.
- The table below contains design parameters for all the sections of V-belt

Section	kW range	Minimum pulley pitch diameter (mm)	Width (mm)	Thickness (mm)
A	0.4 - 4	125	13	8
B	1.5 -15	200	17	11
C	10 -70	300	22	14
D	35-150	500	32	19
E	70-260	630	38	23

Belt Section Selection

- Belt section selection depends solely on the power transmission required, irrespective of number of belts.
- If the required power transmission falls in the overlapping zone, then one has to justify the selection from the economic view point also.
- In general, it is better to choose that section for which the required power transmission falls in the lower side of the given range.
- Another restriction of choice of belt section arises from the view point of minimum pulley diameter.
- If a belt of higher thickness (higher section) is used with a relatively smaller pulley, then the bending stress on the belt will increase, thereby shortening the belt life.

V-Belt Designation

- V-belts are designated with nominal inside length (this is easily measurable compared to pitch length).
- **Inside length + X=Pitch Length**

Value Of X

	A	B	C	D
X (mm)	36	43	56	79

- A **B- section belt** with nominal inside length of 1016 mm or 40 inches is designated as.

B 1016/40

(mm) (inches)

V-Belt Equation

- V-belts have additional friction grip due to the presence of wedge. Therefore, modification is needed in the equation for belt tension. The equation

$$\frac{T_1 - mv^2}{T_2 - mv^2} = e^{\mu\alpha / \sin\frac{\theta}{2}}$$

V-Belt power rating

- Each type of belt section has a power rating. The power rating is given for different pitch diameter of the pulley and different pulley speeds for an angle of wrap of 180^0. A typical nature of the chart is shown below.

kW rating of V-belts for different belt speeds ($\alpha = 180^o$)

Belt Section	Pitch Diameter	N_1	N_2	N_3	N_4
A	D_1	kW_1	kW_2	kW_3	kW_4
	D_2				
	D_3				

V-Belt design factors

- Service factor-The service factor depends on hours of running, type of shock load expected and nature of duty.

Design Power (P_D) = service factor (C_s)* Required Power (P)

C_s = 1.1 to 1.8 for light to heavy shock.

The power rating of V-belt is estimated based on the equivalent smaller pulley diameter (d_{ES}).

$$d_E = C_{SR}\, d$$

C_{SR} depends on the speed ratio

V-Belt design factors

- Angle of wrap correction factor-The power rating of V-belts are based on angle of wrap, **α =**180^0 . Hence, Angle of wrap correction factor ($\mathbf{C_{vw}}$) is incorporated when α is not equal to 180^o .

- *Belt length correction factor* -There is an optimum belt length for which the power rating of a V-belt is given. Depending upon the amount of flexing in the belt in a given time a belt length correction factor ($\mathbf{C_{vL}}$) is used in modifying power rating.

Modified Power Rating

- Therefore, incorporating the correction factors,
- **Modified power rating of a belt (kW)**

  = Power rating of a belt (kW) x C_{vw} x C_{vl}

Selection of V-Belt

- The transmission ratio of V belt drive is chosen within a range of 1:15
- Depending on the power to be transmitted a convenient V-belt section is selected.
- The belt speed of a V-belt drive should be around 20m/s to 25 m/s, but should not exceed 30 m/s.
- From the speed ratio, and chosen belt speed, pulley diameters are to be selected from the standard sizes available.
- Depending on available space the center distance is selected, however, as a guideline,

$$D < C < 3(D + d)$$

Selection of V-Belt

- The belt pitch length can be calculated if C, d and D are known. Corresponding inside length then can be obtained from the given belt geometry. Nearest standard length, selected from the design table, is the required belt length.

- The design power and modified power rating of a belt can be obtained using the equations. Then

$$\text{Number of belts} = \frac{\text{Design Power}}{\text{Modified power rating of the belt}}$$

GEARS

GEARS

- Gears are machine elements used to transmit rotary motion between two shafts, normally with a constant ratio.
- In practice the action of gears in transmitting motion is a cam action, each pair of mating teeth acting as cams.
- The smaller gear in a pair is often called the *pinion*; the larger, either the *gear*, or the *wheel*.

GEAR CLASSIFICATION

- Gears may be classified according to the relative position of the axes of revolution
- The axes may be

    1.parallel,

    2.intersecting,

    3.neither parallel nor intersecting.

GEARS FOR CONNECTING PARALLEL SHAFTS

- *Spur gears*
- *Parallel helical gears*
- *Herringbone gears* (or double-helical gears)
- *Rack* and *pinion* (The rack is like a gear whose axis is at infinity.)

GEARS FOR CONNECTING INTERSECTING SHAFTS

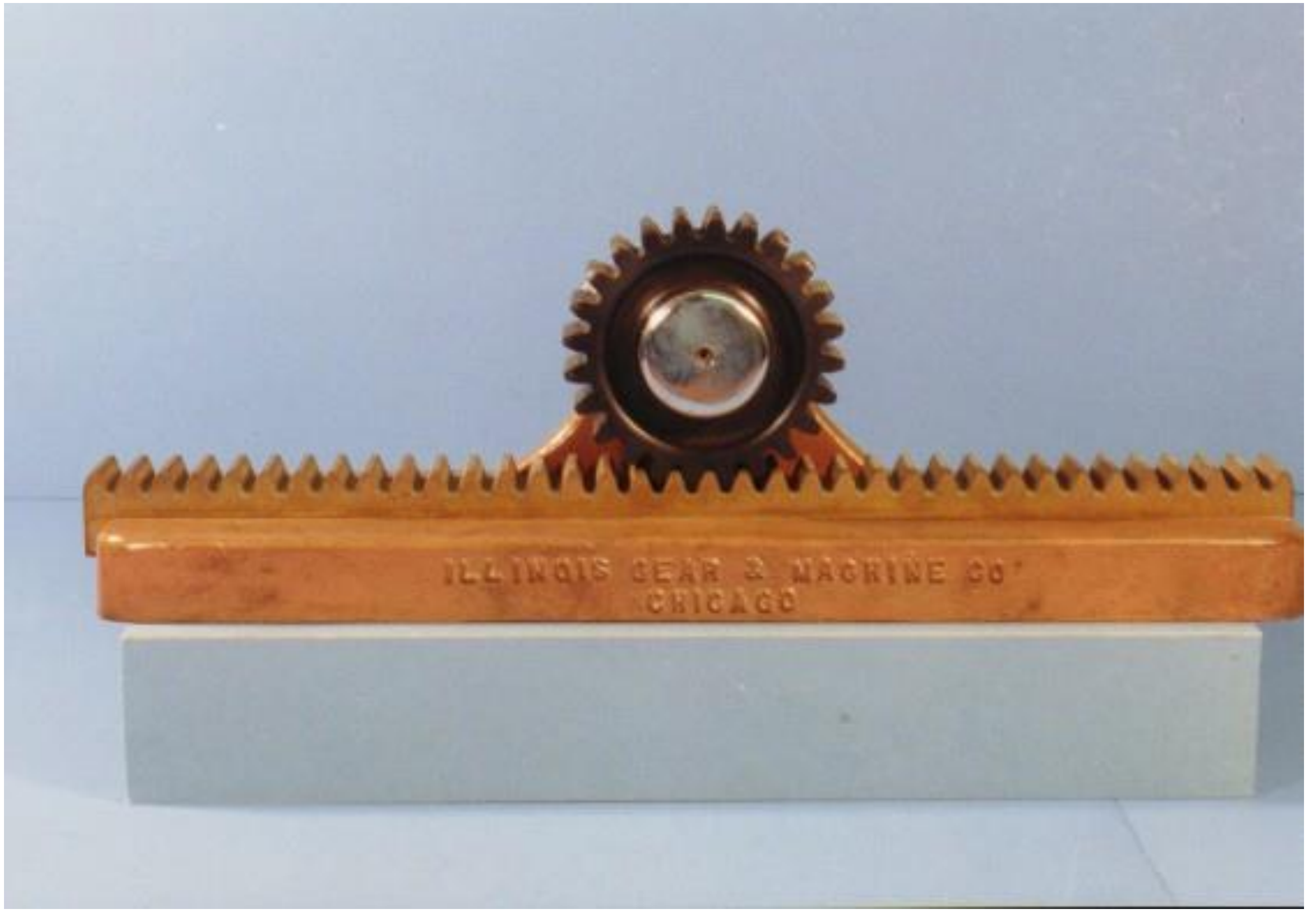
- *Straight bevel gears*
- *Spiral bevel gears*

NEITHER PARALLEL NOR INTERSECTING SHAFTS

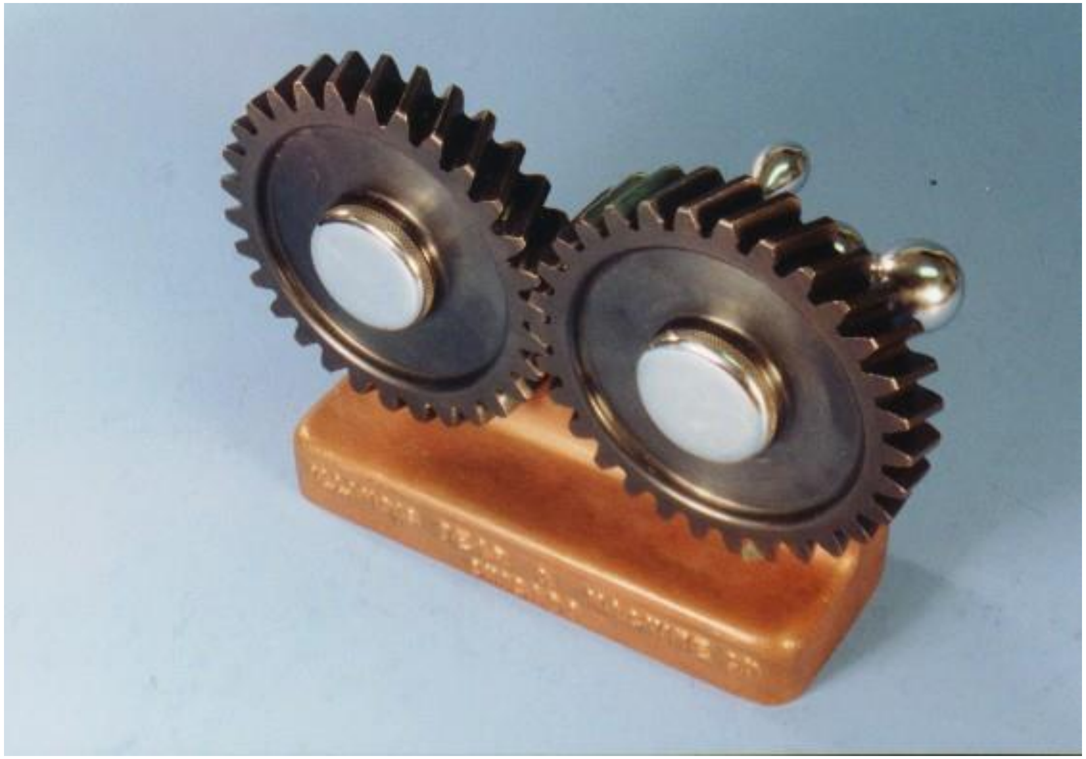
- *Crossed-helical gears*
- *Hypoid gears*
- *Worm and worm gear*

SPUR GEAR PAIR

SPUR RACK AND PINION

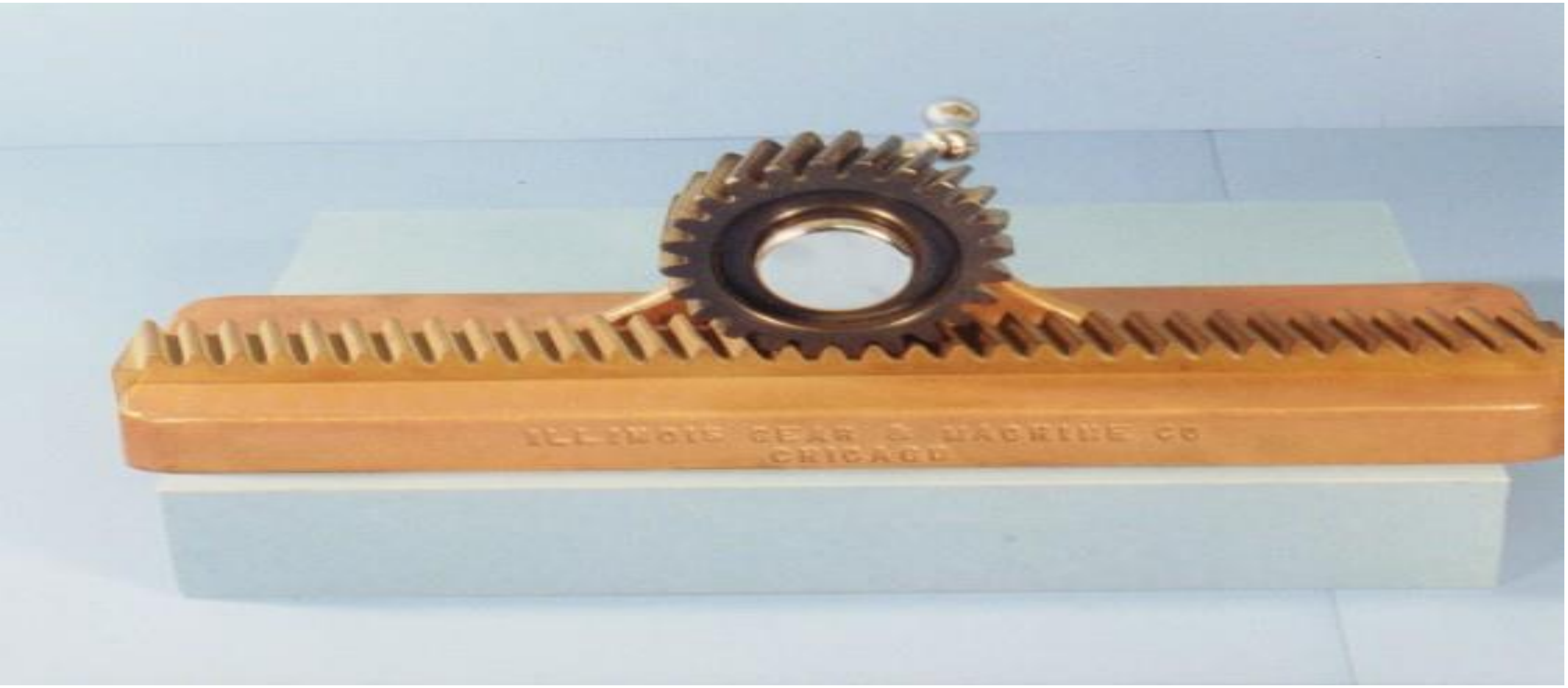


HELICAL GEAR PAIR



HELICAL GEARS

HELICAL RACK AND PINION



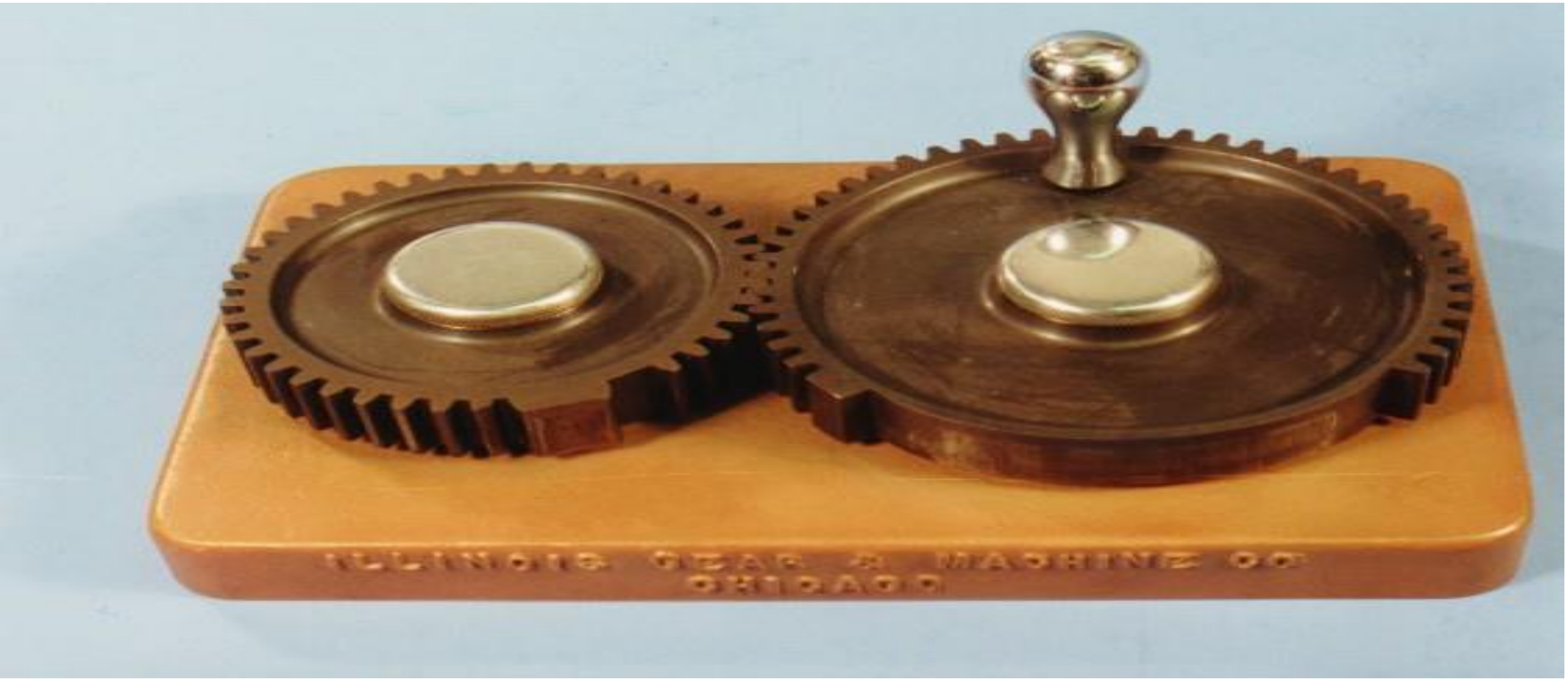
ECCENTRIC SPUR GEAR PAIR

HERRINGBONE GEAR PAIR

DOUBLE HELICAL GEAR PAIR

SINGLE PLANETORY GEAR SET

INTERMITTENT SPUR GEAR PAIR



RIGHT ANGLE STRAIGHT BEVEL PAIR

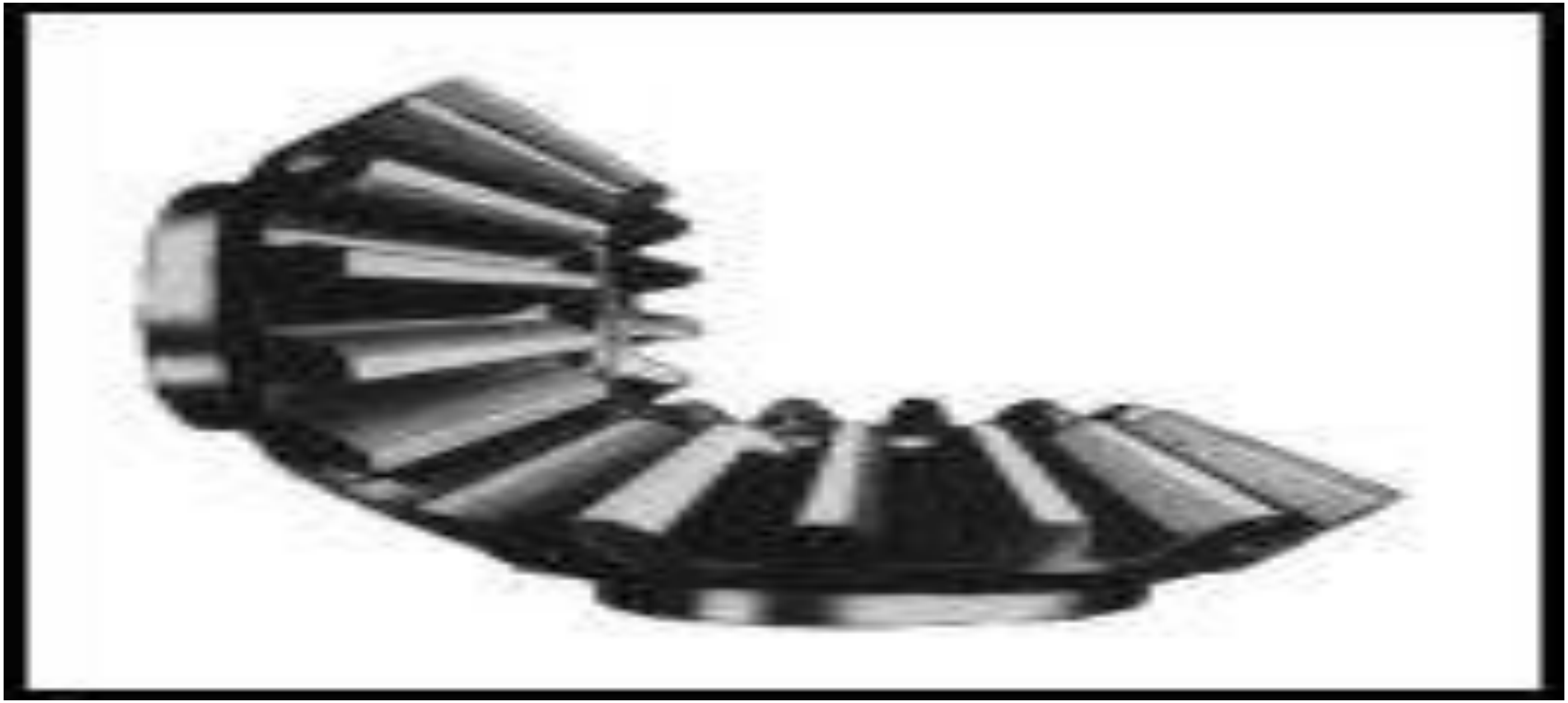
RIGHT ANGLE STRAIGHT BEVEL PAIR

NON-RIGHT ANGLE STRAIGHT BEVEL PAIR

NON-RIGHT ANGLE STRAIGHT BEVEL PAIR

SPIRAL BEVEL GEAR PAIR

MITER GEAR



SPIRAL BEVEL GEAR PAIR

HYPOID BEVEL GEAR PAIR

RIGHT ANGLE WORM GEAR PAIR

NON-RIGHT ANGLE WORM GEAR PAIR

LAW OF GEARING

AS THE GEAR ROTATE,THE COMMON NORMAL AT THE POINT OF CONTACT BETWEEN THE TEETH ALWAYS PASS THROUGH A FIXED POINT ON THE LINE OF CENTERS.THE FIXED POINT IS CALLED PITCH POINT.IF THE TWO GEARS IN MESH SATISFY THE BASIC LAW, THE GEARS ARE SAID TO PRODUCE CONJUGATE ACTION.

LAW OF GEARING

PITCH LINE VELOCITY, $V = \omega_1 r_1 = \omega_2 r_2$

- SPEED RATIO,

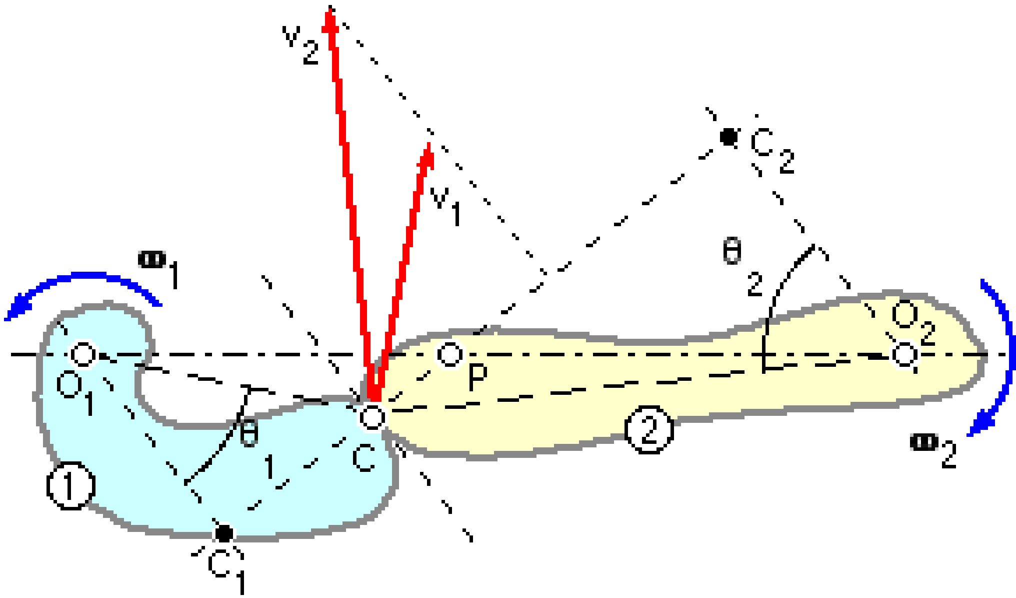
$$r_{s=} \omega_2/\omega_1 = N_2/N_1 = Z_1/Z_2 = d_1/d_2$$

ω -angular velocity, N-speed, Z- no of teeth

d-pitch circle diameter

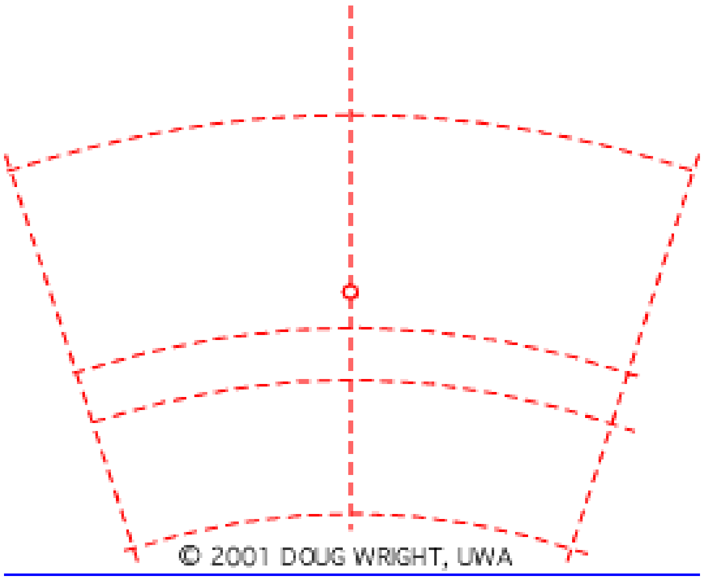
CONJUGATE ACTION

- It is essential for correctly meshing gears, the size of the teeth (the module) must be the same for both the gears.
- Another requirement - the shape of teeth necessary for the speed ratio to remain constant during an increment of rotation; this behavior of the contacting surfaces (ie. the teeth flanks) is known as ***conjugate action.***



TOOTH PROFILES

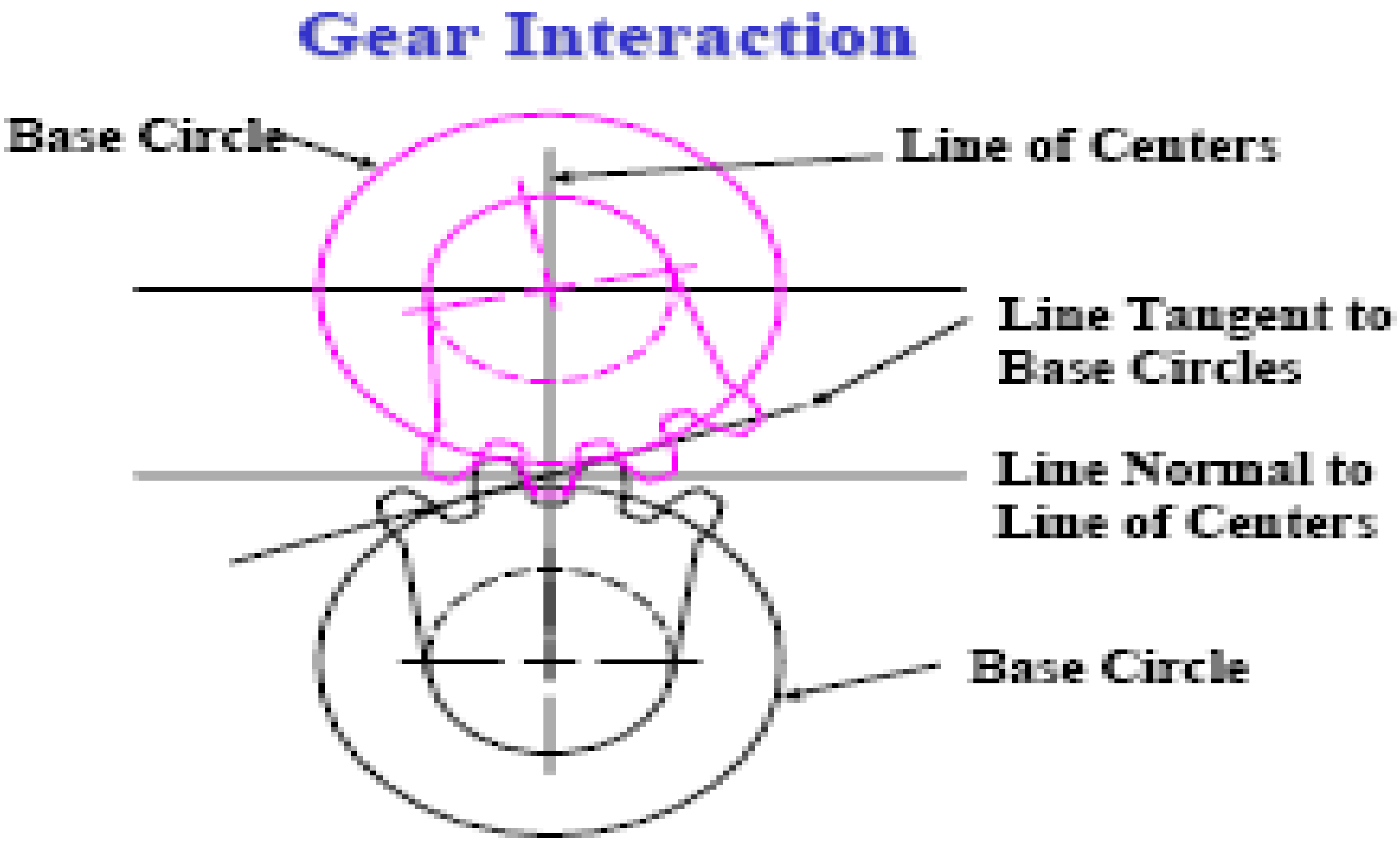
- Many tooth shapes are possible for which a mating tooth could be designed to satisfy the fundamental law, only two are in general use: the *cycloidal* and *involute* profiles.
- Modern gearing (except for clock gears) is based on involute teeth.



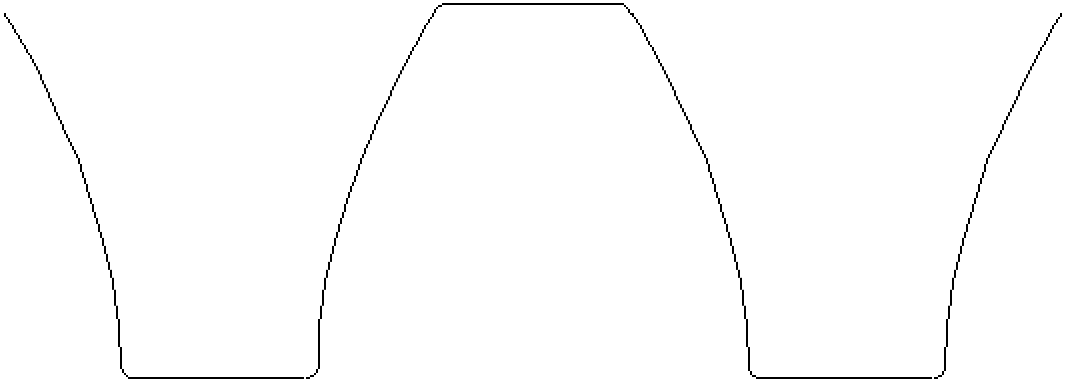
TOOTH PROFILES

- The involute tooth has great advantages in ease of manufacture, interchangeability, and variability of centre-to-centre distances.
- Cycloidal gears still have a few applications - these may be used in mechanical clocks, the slow speeds and light loads in clocks do not require conjugate gear tooth profiles.

TOOTH PROFILES

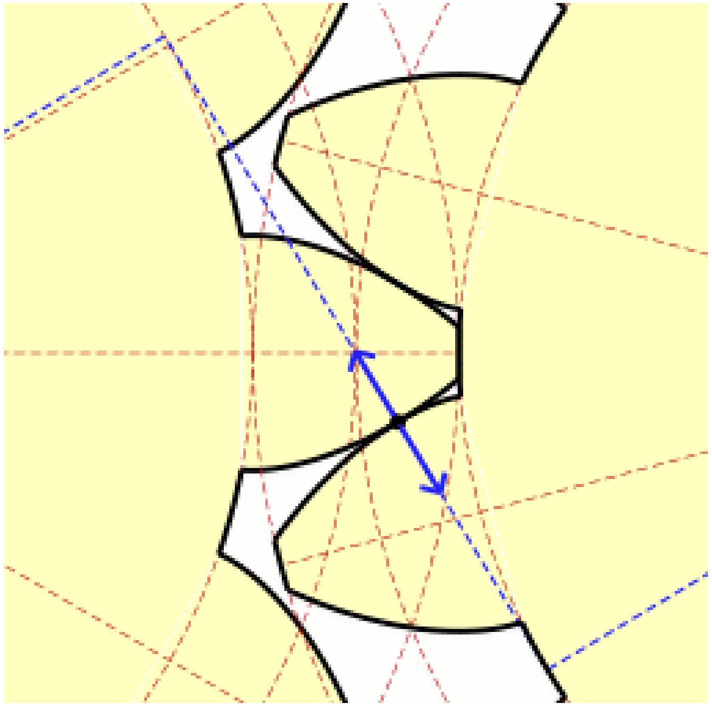


INVOLUTE PROFILES



INVOLUTE PROFILES

Blue arrows shows the contact forces. The force line runs along common tangent to base circles.



ADVANTAGES OF INVOLUTE PROFILE

It is easy to manufacture and the center distance between a pair of involute gears can be varied without changing the velocity ratio. Thus close tolerances between shaft locations are not required. The most commonly used *conjugate* tooth curve is the *involute curve.* (Erdman & Sandor).

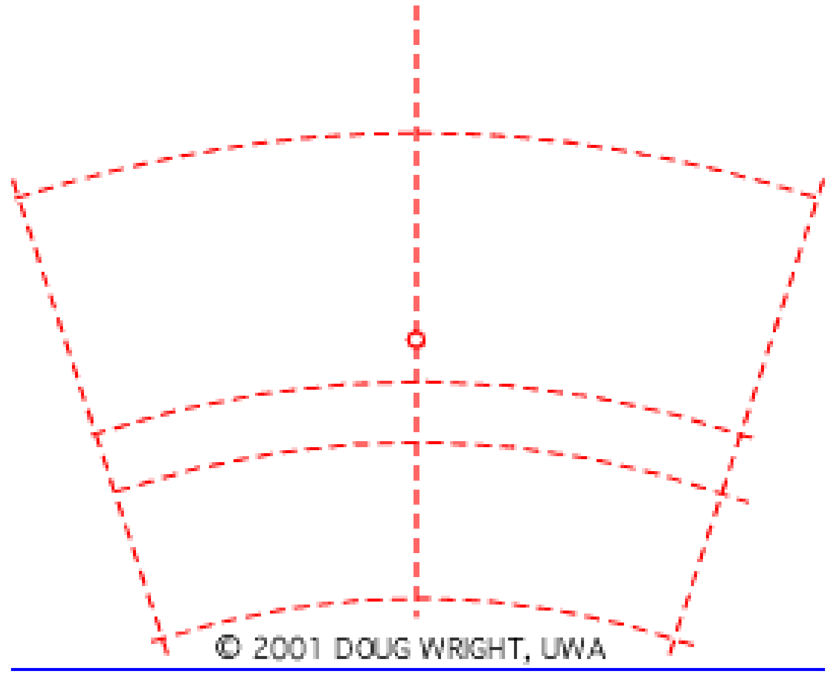
Advantages of involute gears

- In involute gears, the pressure angle, remains constant between the point of tooth engagement and disengagement. It is necessary for smooth running and less wear of gears.
- But in cycloidal gears, the pressure angle is maximum at the beginning of engagement, reduces to zero at pitch point, starts increasing and again becomes maximum at the end of engagement. This results in less smooth running of gears.

ADVANTAGES OF INVOLUTE PROFILES

- The face and flank of involute teeth are generated by a single curve where as in cycloidal gears, double curves (i.e. epi-cycloid and hypo-cycloid) are required for the face and flank respectively. Thus the involute teeth are easy to manufacture than cycloidal teeth.
- In involute system, the basic rack has straight teeth and the same can be cut with simple tools.

GEAR TOOTH GENERATOR



ADVANTAGES OF CYCLOIDAL PROFILES

Since the cycloidal teeth have wider flanks, therefore the cycloidal gears are stronger than the involute gears, for the same pitch. Due to this reason, the cycloidal teeth are preferred specially for cast teeth.

ADVANTAGES OF CYCLOIDAL PROFILES

In cycloidal gears, the contact takes place between a convex flank and a concave surface, where as in involute gears the convex surfaces are in contact. This condition results in less wear in cycloidal gears as compared to involute gears. However the difference in wear is negligible.

ADVANTAGES OF CYCLOIDAL PROFILES

In cycloidal gears, the interference does not occur at all. Though there are advantages of cycloidal gears but they are outweighed by the greater simplicity and flexibility of the involute gears.

SYSTEMS OF GEAR TEETH

The following four systems of gear teeth are commonly used in practice:

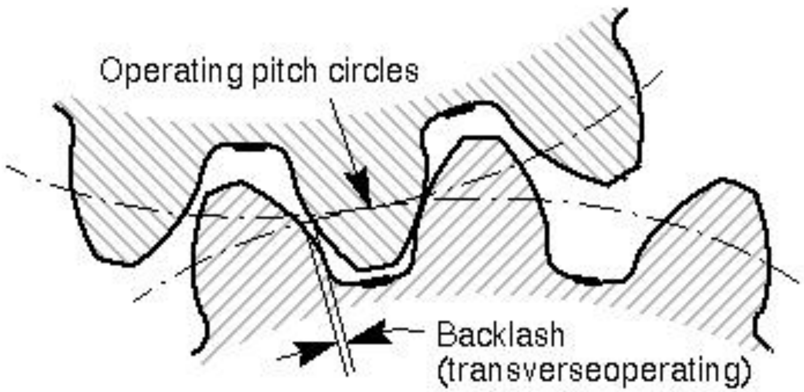
1. 14 ½^0 Composite system
2. 14 ½^0 Full depth involute system
3. 20^0 Full depth involute system
4. 20^0 Stub involute system

SYSTEMS OF GEAR TEETH

- The $14½^0$ *composite system* is used for general purpose gears.
- It is stronger but has no interchangeability. The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion.
- The teeth are produced by formed milling cutters or hobs.
- The tooth profile of the $14½^0$ *full depth involute system* was developed using gear hobs for spur and helical gears.

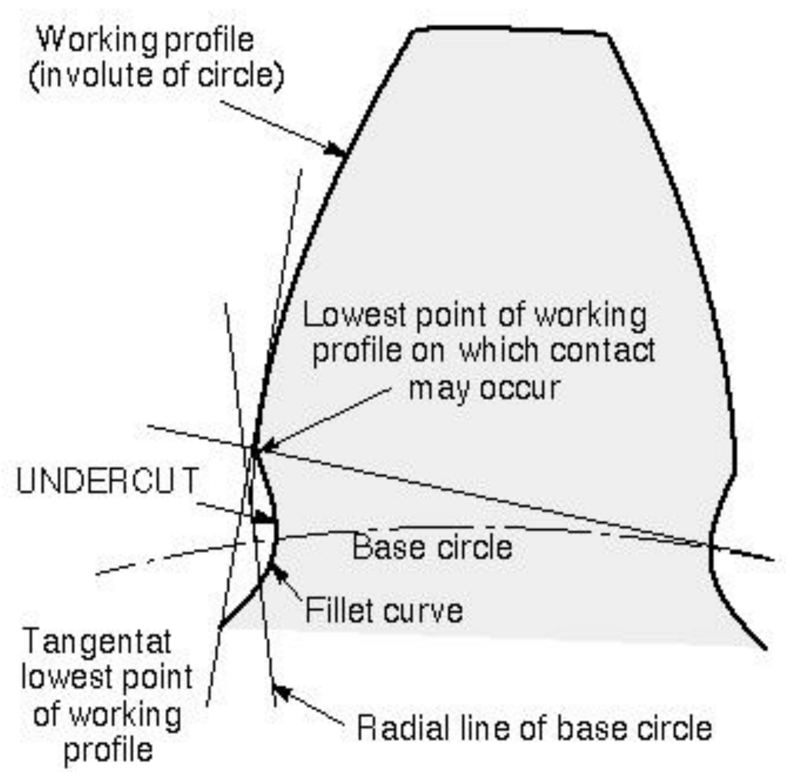
SYSTEMS OF GEAR TEETH

- The tooth profile of the 20^0 *full depth involute system* may be cut by hobs.
- The increase of the pressure angle from $14½^0$ to 20^0 results in a stronger tooth, because the tooth acting as a beam is wider at the base.
- .In the stub tooth system, the tooth has less working depth-usually 20% less than the full depth system. The addendum is made shorter.
- The *20^0 stub involute system* has a strong tooth to take heavy loads



- Backlash is the error in motion that occurs when gears change direction. It exists because there is always some gap between the tailing face of the driving tooth and the leading face of the tooth behind it on the driven gear, and that gap must be closed before force can be transferred in the new direction

UNDERCUT



- Undercut is a condition in generated gear teeth when any part of the fillet curve lies inside of a line drawn tangent to the working profile at its point of juncture with the fillet

INTERFERNCE

- When two gears are in mesh at one instant there is a chance to mate involute portion with non-involute portion of mating gear. This phenomenon is known as **INTERFERENCE** and occurs when the number of teeth on the smaller of the two meshing gears is smaller than a required minimum. To avoid interference we can have 'undercutting', but this is not a suitable solution as undercutting leads to weakening of tooth at its base. In this situation the minimum number of teeth on pinion are specified, which will mesh with any gear,even with rack, without interference.

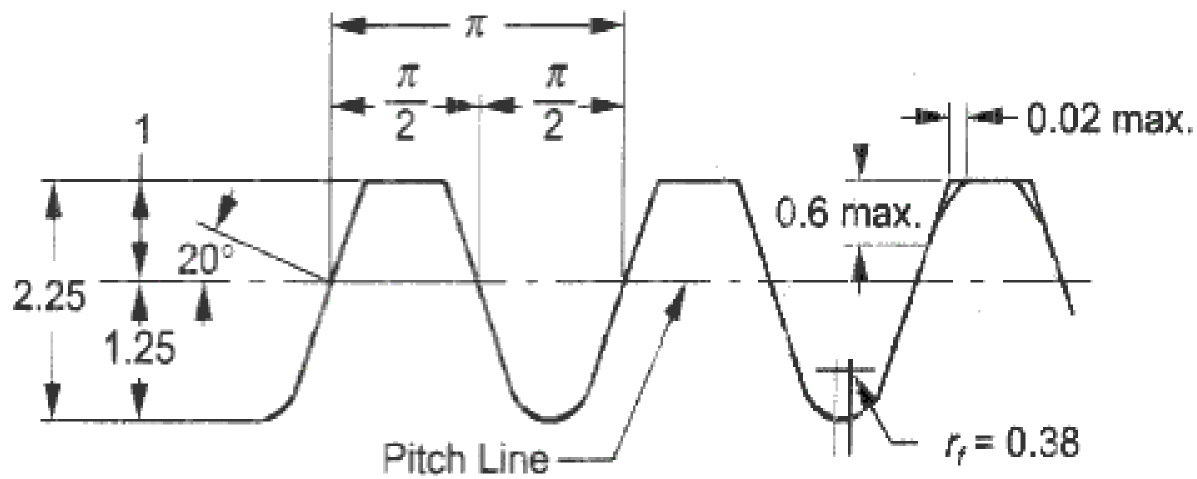
MODULE AND DIAMETRAL PITCH

The gear proportions are based on the module.

- m = (Pitch Circle Diameter(mm)) / (Number of teeth on gear).
- In the USA the module is not used and instead the Diametral Pitch (P) is used

P = (Number of Teeth) / Pitch Circle Diameter(mm)

GEAR PROPORTIONS



Profile of a standard 1mm module gear teeth for a gear with Infinite radius (Rack). Other module teeth profiles are directly proportion . e.g. 2mm module teeth are 2 x this profile

SPUR GEAR DESIGN

- The spur gear is the simplest type of gear manufactured and is generally used for transmission of rotary motion between parallel shafts. The spur gear is the first choice option for gears except when high speeds, loads, and ratios direct towards other options. Other gear types may also be preferred to provide more silent low-vibration operation.
- A single spur gear is generally selected to have a ratio range of between 1:1 and 1:6 with a pitch line velocity up to 25 m/s. The spur gear has an operating efficiency of 98-99%. The pinion is made from a harder material than the wheel.
- A gear pair should be selected to have the highest number of teeth consistent with a suitable safety margin in strength and wear.

STANDARD PROPORTIONS

. American Standard Association (ASA)

. American gear Manufacturers Association (AGMA)

. Brown and Sharp

. 14 ½ deg,20deg, 25deg pressure angle

. Full depth and stub tooth systems

GEAR MATERIALS

The gear materials should have the following properties.

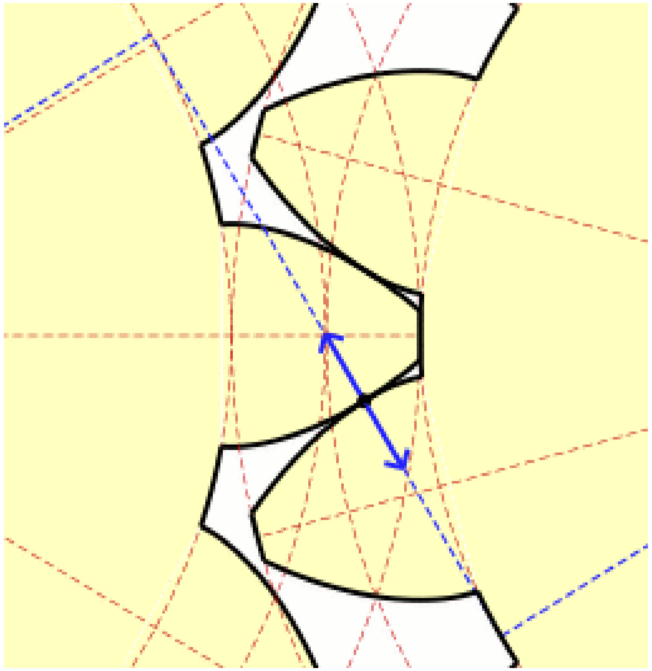
1. High tensile strength to prevent failure against static loads
2. High endurance strength to withstand dynamic loads
3. Good wear resistance to prevent failure due to contact stresses which cause pitting and scoring.
4. Low coefficient of friction
5. Good manufacturability

GEAR MATERIALS

1. Ferrous Metals- Cast iron, Cast Steel, Alloy Steel, Plain Carbon Steel, Stainless steel.
2. Non-Ferrous Metals- Aluminium alloys, Brass alloys, Bronze alloys,Megnesium alloys, Nickel alloys, Titanium alloys, Die cast alloys, Sintered Powdered alloys.
3. Non metals- Acetal, Phenolic laminates, Nylons, PTFE

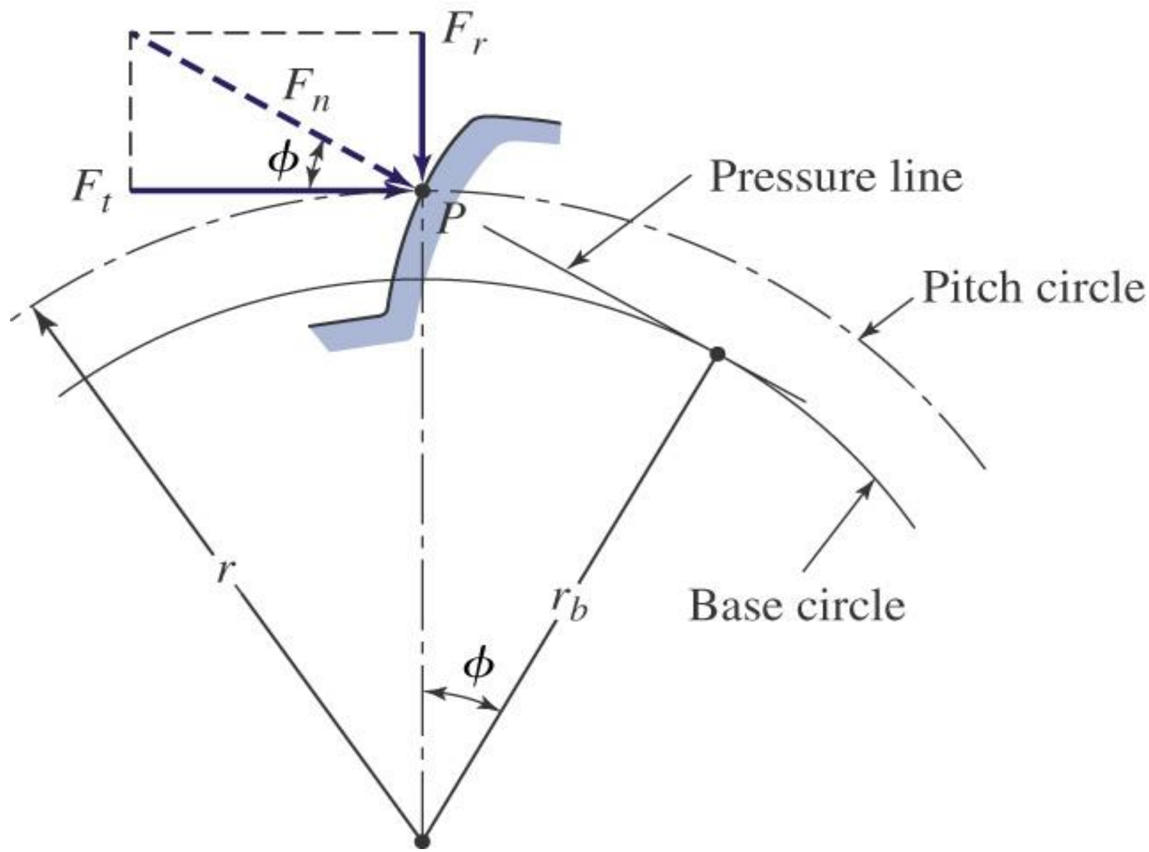
FORCES ON SPUR GEAR TEETH

Blue arrows shows the contact forces. The force line runs along common tangent to base circles.



Transmitted Load

- With a pair of gears or gear sets, Power is transmitted by the force developed between contacting Teeth



FORCES ON SPUR GEAR TEETH

The spur gear's transmission force F_n, which is normal to the tooth surface can be resolved into a tangential component, F_t, and a radial component, F_r .

The effect of the tangential component is to produce maximum bending at the base of the tooth.

The effect of the radial component is to produce uniform compressive stress over the cross- section, which is small compared to bending stress; and so usually neglected.

FORCES ON SPUR GEAR TEETH

Tangential force on gears $F_t = F_n \cos \alpha$

Separating force on gears $F_r = F_n \sin \alpha = F_t \tan \alpha$

Torque on driver gear $(M_t)_1 = F_t\, d_1 / 2$

Torque on driven gear $(M_t)_2 = F_t\, d_2 / 2$

SURFACE SPEED

Surface speed is also referred to as pitch line speed (v)

$$v = \pi \, d \, N / 60 \quad m/s$$

d- pitch circle diameter

N- Speed

POWER TRANSMITTED

The power transmitting capacity of the spur gear (P) is

$$P = F_t \, v / 1000 \qquad \text{kW}$$

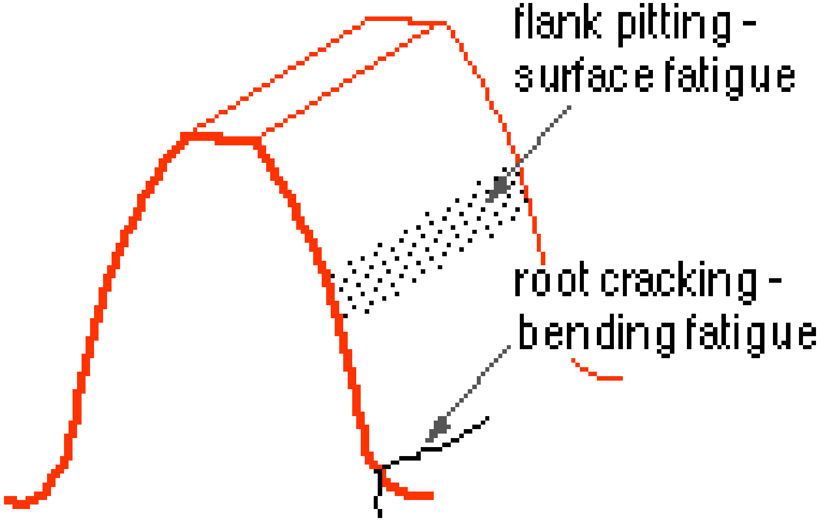
MODES OF GEAR FAILURE

Apart from failiure-off overloads, there are three common modes of tooth failure

.bending fatigue leading to root cracking,

.surface contact fatigue leading to flank***pitting,*** and

.lubrication breakdown leading to ***scuffing.***



SPUR GEAR STRENGTH

Bending strength –Lewis Formula

$$F_t = \sigma\, b\, y\, p$$

σ- induced bending stress

b – face width

p- circular pitch = Π m

y- Lewis form factor

SPUR GEAR STRENGTH

Bending strength –Lewis Formula

$$\mathbf{F_t = \sigma\, b\, Y\, m}$$

σ- induced bending stress

b – face width

m- module

Y- Form factor, $Y = \pi y$

Lewis form factor

- $y = 0.124 – 0.684/z$ for $14\frac{1}{2}^{o}$ involute system.
- $y = 0.154 - 0.912/z$ for 20^{0} involute system.
- $y = 0.175 - 0.841/z$ for 20^{o} stub involute system.

VELOCITY FACTOR

When a gear wheel is rotating the gear teeth come into contact with some degree of impact. To allow for this a velocity factor

(C_V) is introduced into the Lewis equation so that

$\sigma = C_V \sigma_d$

where σ_{d} - Allowable static stress for the gear material,

$$\mathbf{F_t = \sigma_d C_v b Y m}$$

Design considerations

- The load carrying capacity of a gearing unit depends upon the weaker element on it.

- When both the gear and the pinion are made of the same material, the pinion becomes the weaker element.

Design considerations

- When the pinion and the gear are made of different materials, the product σ_d y, known as the strength factor, determines the weaker element.
- That is, the element for which the product is less, is the weaker one.

Design considerations

- The number of teeth on the pinion is less than that on the gear. Hence, $y_P < y_{G.}$
- The condition for the equality of the strength for the pinion and gear is
- $$(\sigma_d\, y)_P = (\sigma_d\, y)_G$$
- From the above, it is evident that the pinion must be made of a stronger material.

STRESS CONCENTRATION FACTOR

There is a concentration of stress where the tooth joins the bottom land. This causes the actual stress to be greater than that will obtain from the Lewis equation for a given tangential load. The effect of stress concentration is considered by incorporating a stress concentration factor, K_C in calculating the Lewis equation ,

$$\mathbf{F_t = \sigma_d C_v b Y m / K_c}$$

SERVICE FACTOR /OVERLOAD FACTOR

The value of the tangential load $\mathbf{F_t}$ to be used in the Lewis equation is the load for which the drive is to be designed and includes the service conditions of the drive. It may be obtained from the power equation,

$$\mathbf{F_t = 1000\ P\ C_s\ /v,}$$

where P= Power to be transmitted or rated power of the prime mover, kW,

C_s = service factor and v = pitch line velocity, m/s

Beam strength

- The strength of the tooth determined from the Lewis equation, based on the static strength of the weaker material, is known as the beam strength, F_{en} of the tooth. It is given by,

$$\mathbf{F_{en} = \sigma_{en}\, b\, y\, p = \sigma_{en}\, b\, Y\, m}$$

σ_{en} = endurance limit stress.

Dynamic loading

The contributory factor for dynamic loading

- Inaccuracies in teeth spacing.
- Elements of teeth surfaces, not parallel to the gear axis.
- Deflection of teeth under load.
- Deflection and twisting of shaft under load.

Dynamic loading

- In Lewis equation the velocity factor was considered to account for the effect of dynamic loading to a limited extent.

- The dynamic load on the gear tooth will be greater than the steady transmitted load, F_t.

BUCKINGHAM'S EQUATION FOR DYNAMIC LOAD

The maximum instantaneous load on the tooth, known as dynamic load is given by Buckingham as follows:

Dynamic load (F_d) =Tangential tooth load (F_t) + Increment load due to dynamic action (F_i)

$$\mathbf{F_i = 20.67\ v\ (C\ b + F_t)\ /\ [20.67\ v + (C\ b + F_t)^{1/2}]}$$

Where **C** is called **dynamic factor** depending upon the machining errors.

Design considerations

- The gear tooth section, determined by using Lewis equation, must be checked for dynamic and wear loading.

DYNAMIC STRENGTH

- **Dynamic strength** of a gear is the maximum load that a gear can support and is obtained, from the Lewis equation by excluding the velocity factor, as

$$\mathbf{F_S = \sigma_d\ b\ Y\ m} \geq F_d$$

WEAR STRENGTH

After checking the dynamic strength of a tooth, it must be checked for wear. The maximum load that a gear tooth can withstand without wear failure depends upon the radius of curvature of the tooth profile, elasticity and the surface endurance limit of the material. According to Buckingham the **wear strength** is given as,

$$\mathbf{F_w = d_1\, b\, Q\, K \qquad \geq F_d}$$

Where d_1 = pitch diameter of pinion, b =face width,

Q = ratio factor = $2\, d_2 / (d_2 + d_1) = 2\, z_2 / (z_2 + z_1)$

K = load stress factor.

Design Considerations

- The gear should have sufficient strength so that it does not fail under static and dynamic loading.
- The gear teeth must have good wear characteristics.
- Suitable material combination must be chosen for the gearing.
- The drive should be compact.

Design of Helical Gears

What is the purpose of the brake?

BRAKES

- A brake is a device by means of which artificial resistance is applied on to a moving machine member in order to retard or stop the motion of the member or machine

- **Types of Brakes**

Based on the working principle used brakes can be classified as mechanical brakes, hydraulic brakes, electrical (eddy current) magnetic and electro-magnetic types.

MECHANICAL BRAKES

- Mechanical brakes are invariably based on the frictional resistance principles• In mechanical brakes artificial resistances created using frictional contact between the moving member and a stationary member, to retard or stop the motion of the moving member.

According to the direction of acting force

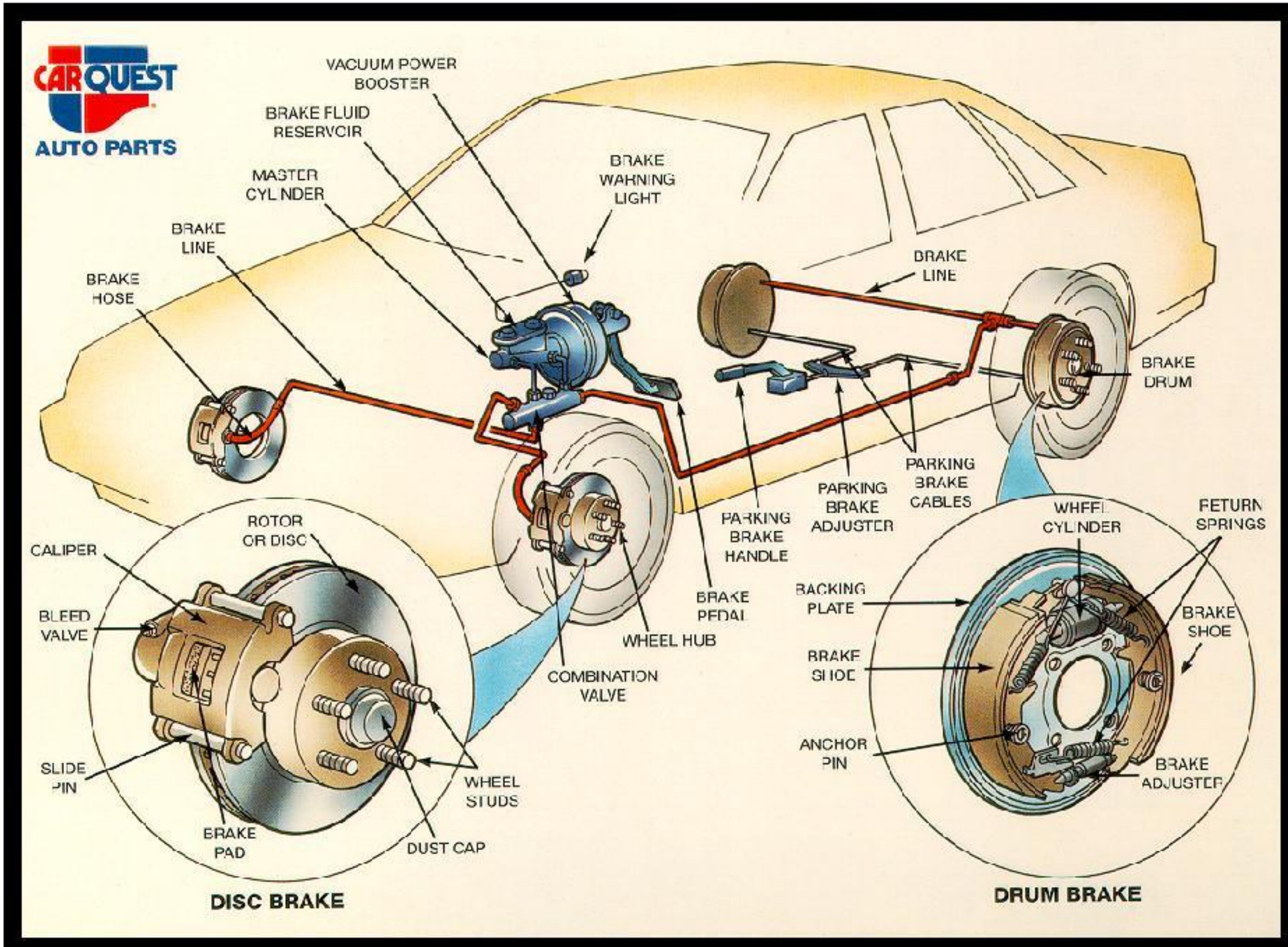
1. Radial brakes
2. Axial brakes

Axial brakes include disc brakes and cone brakes

- Depending upon the shape of the friction elements, radial brakes may be named as
  1. Drum or Shoe brakes
  2. Band brakes
- Many cars have drum brakes on the rear wheels and disc brakes on the front

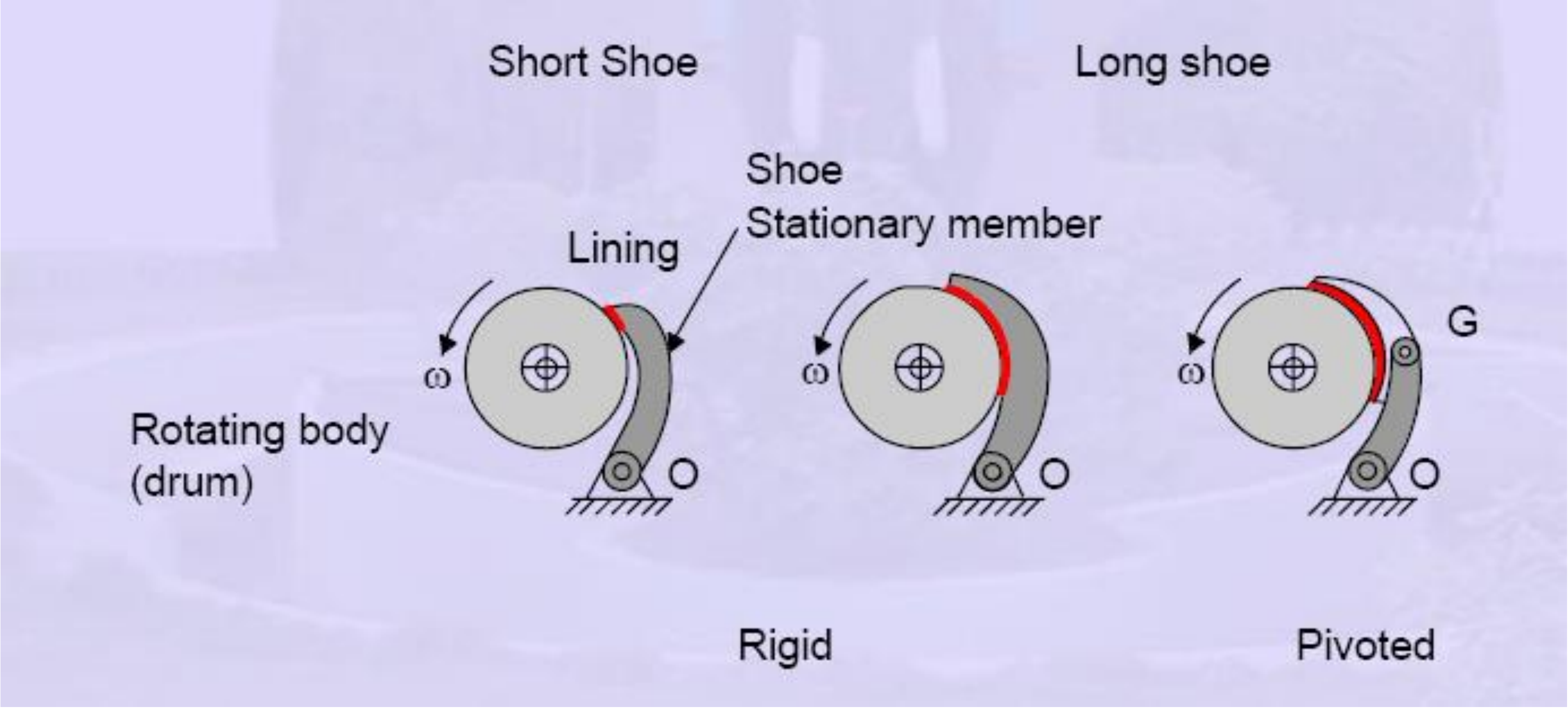
Disc brakes

Drum brake


CARQUEST
AUTO PARTS
VACUUM POWER BOOSTER
BRAKE FLUID RESERVOIR
MASTER CYLINDER
BRAKE WARNING LIGHT
BRAKE LINE
BRAKE HOSE
BRAKE LINE
BRAKE DRUM
PARKING BRAKE CABLES
PARKING BRAKE ADJUSTER
PARKING BRAKE HANDLE
ROTOR OR DISC
CALIPER
BLEED VALVE
SLIDE PIN
BRAKE PAD
DUST CAP
WHEEL STUDS
COMBINATION VALVE
WHEEL HUB
BRAKE PEDAL
WHEEL CYLINDER
RETURN SPRINGS
BACKING PLATE
BRAKE SHOE
BRAKE SHOE
ANCHOR PIN
BRAKE ADJUSTER
DISC BRAKE
DRUM BRAKE

- Drum brakes work on the same principle as disc brakes: **Shoes press against a spinning surface**. In this system, that surface is called a drum.

- Drum Brakes are classified based on the shoe geometry.
- Shoes are classified as being either short or long.
- The shoes are either rigid or pivoted, pivoted shoes are also some times known as hinged shoes

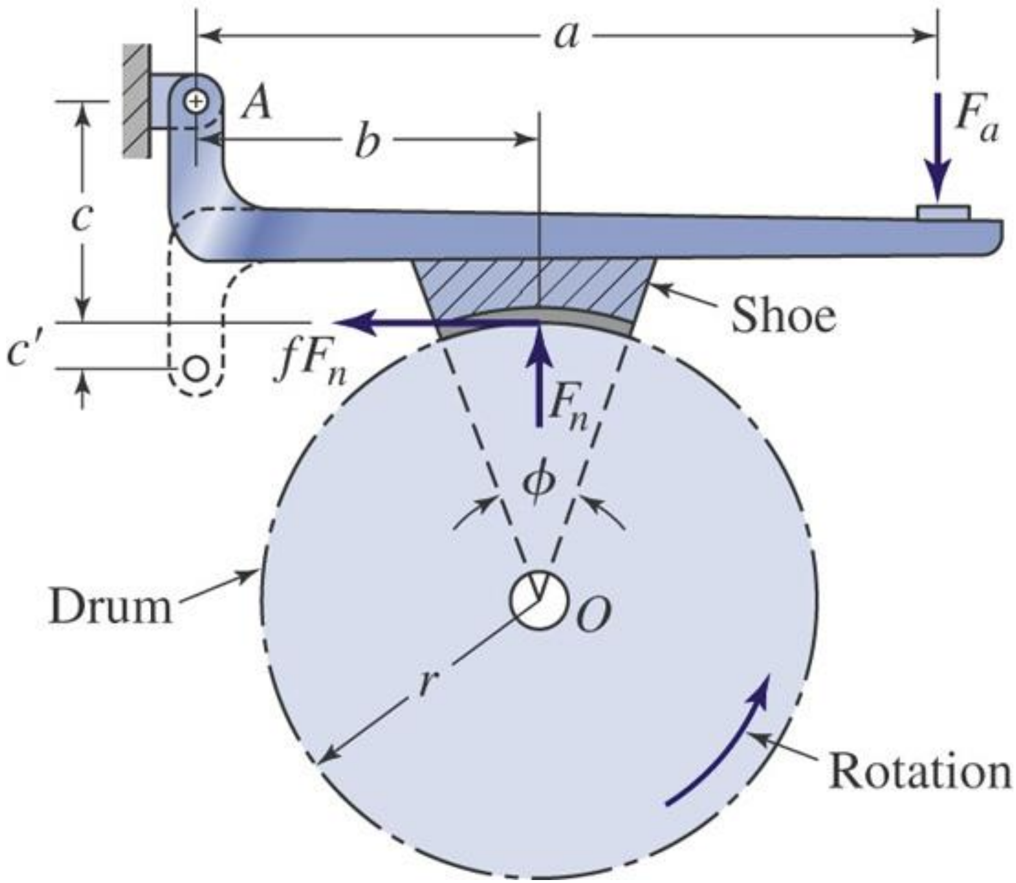

Short Shoe
Long shoe
Shoe
Stationary member
Lining
G
ω
ω
ω
O
O
O
Rotating body
(drum)
Rigid
Pivoted

Design and Analysis

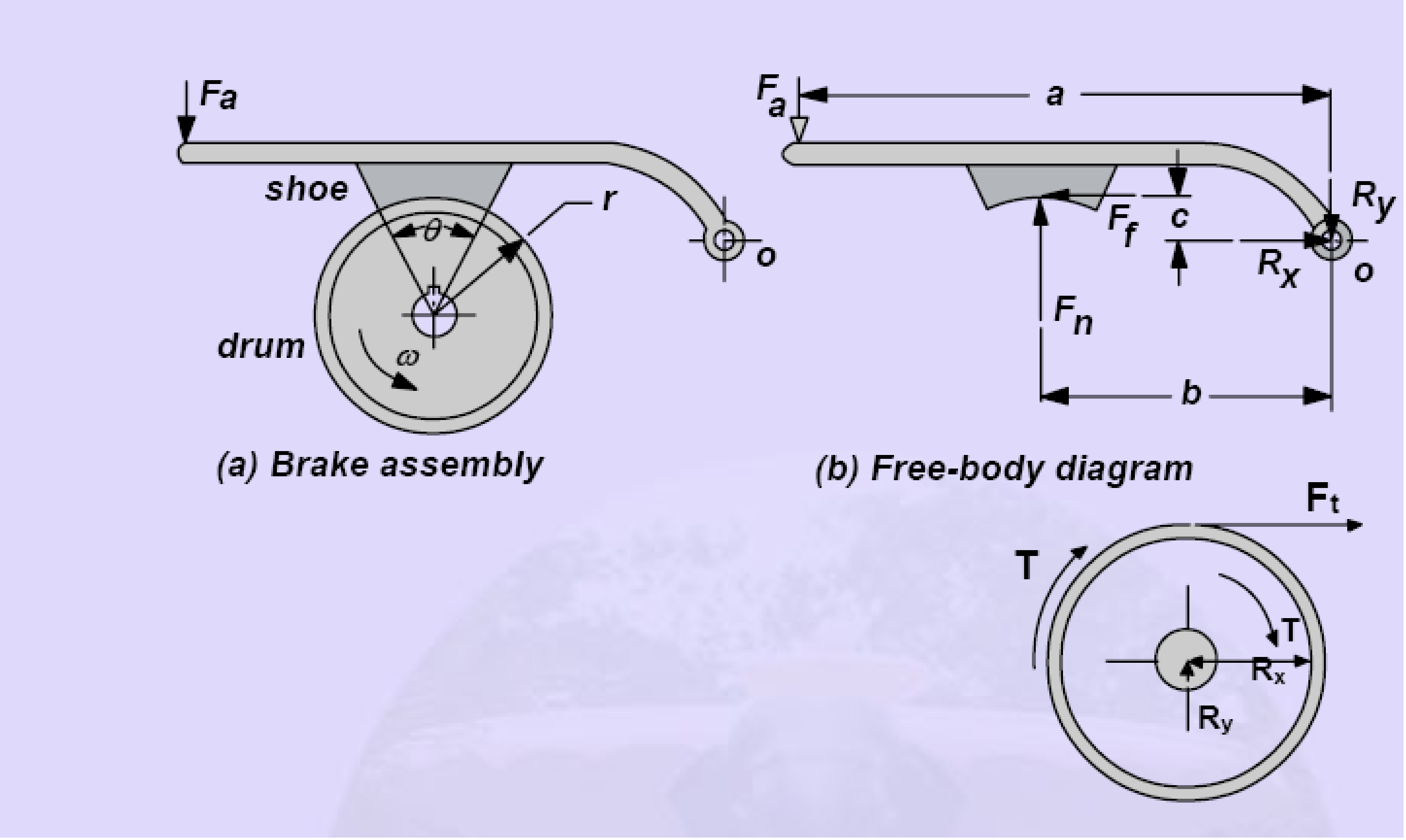
To design, select or analyze the performance of these devices knowledge on the following are required.

- *The braking torque*
- *The actuating force needed*
- *The energy loss and temperature rise*

Short-Shoe Drum Brakes



- On the application of an actuating force F_a, a normal force F_n is created when the shoe contacts the rotating drum. And a frictional force F_f of magnitude $f.F_n$, f being the coefficient of friction, develops between the shoe and the drum.


Fa
shoe
r
θ
o
drum
ω
(a) Brake assembly
Fa
a
Ry
Ff
c
Rx
o
Fn
b
(b) Free-body diagram
Ft
T
T
Rx
Ry

- Moment of this frictional force about the drum center constitutes the braking torque.

The torque on the brake drum is then,

$T = f\,F_n.\,r = f.p.A.r$

- Applying the equilibrium condition by taking moment about the pivot ‘O’ we can write
- Substituti

$$\sum M_O = F_a\, a - F_n\, b + f\, F_n c = 0$$

tuating force, we get,

$$F_a = F_n(b+-fc)/a$$

The reaction forces on the hinge pin (pivot) are found from a summation of forces,

i.e.

$$F_x = 0, R_x = fp_a A$$

$$F_y = 0, R_y = p_a A - F_a$$

Self- energizing

- With the shown direction of the drum rotation (CCW), the moment of the frictional force f. F_n c adds to the moment of the actuating force, F_a
- As a consequence, the required actuation force needed to create a known contact pressure p is much smaller than that if this effect is not present. This phenomenon of frictional force aiding the brake actuation is referred to as *self-energization*.

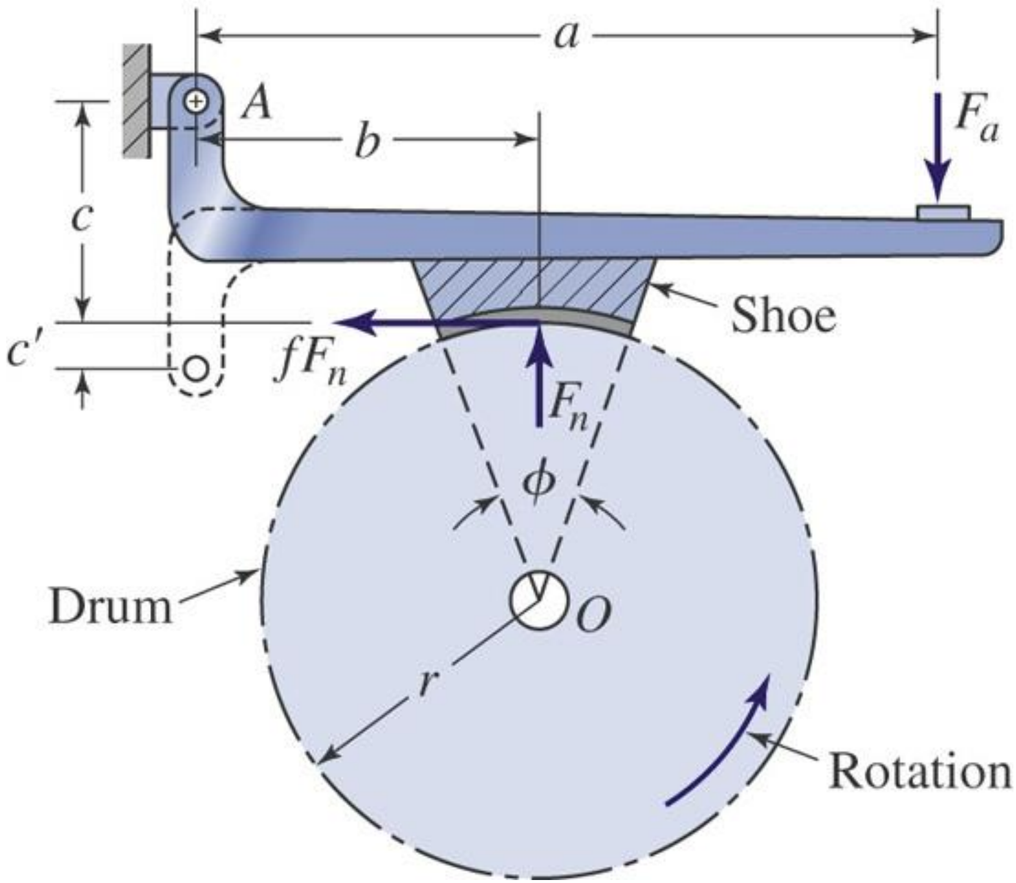
Leading and trailing shoe

- For a given direction of rotation the shoe in which self energization is present is known as the leading shoe
- When the direction of rotation is changed, the moment of frictional force now will be opposing the actuation force and hence greater magnitude of force is needed to create the same contact pressure. The shoe on which self energization is not present is known as a trailing shoe

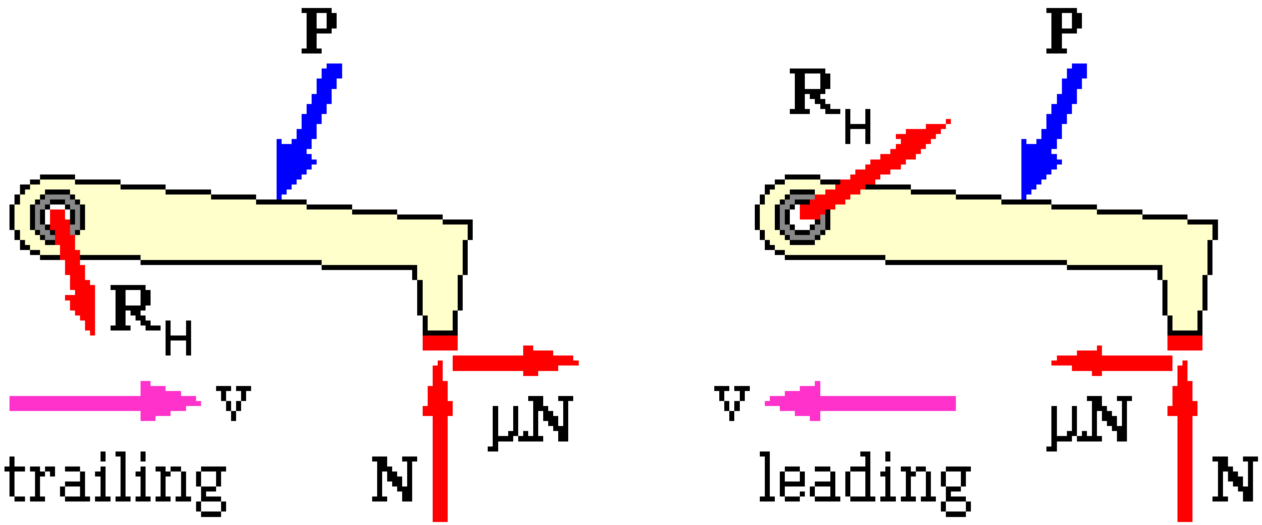
Self Locking

- At certain critical value of f.c the term (b-fc) becomes zero. i.e no actuation force need to be applied for braking. This is the condition for *self-locking*. Self-locking will not occur unless it is specifically desired.

Self-Energizing & Self-Locking Brakes



Self-Energizing & Self-Locking Brakes

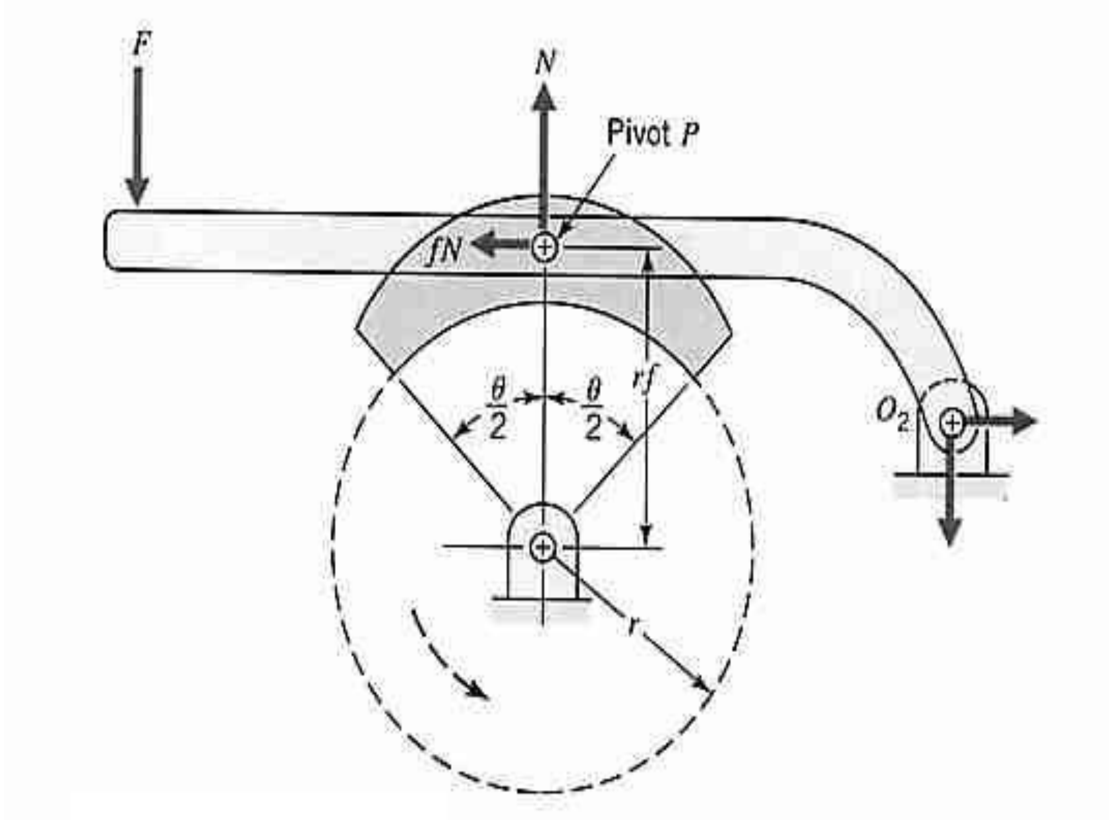


$Pe - Na - \mu Nh = 0$	$Pe - Na + \mu Nh = 0$
$N = Pe / (a + \mu h)$	$N = Pe / (a - \mu h)$
counter-actuating	***self-actuating***

Brake with a pivoted long shoe

- When the shoe is rigidly fixed to the lever, the tendency of the frictional force (f.Fn) is to unseat the block with respect to the lever. This is eliminated in the case of pivoted or hinged shoe brake since the braking force acts through a point which is coincident with the brake shoe pivot.

Pivoted-shoe brake



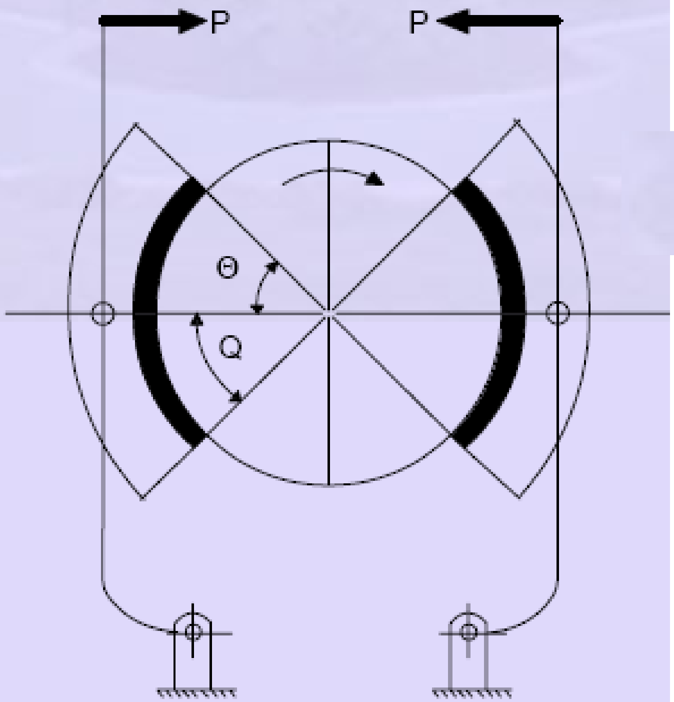
Pivoted-shoe brake

The distance of the pivot from the drum centre

$$r_f = r\frac{4\sin(\theta/2)}{\theta + \sin\theta}$$

Torque transmitted is given by

\DOUBLE BRAKE PIVOTED SHOE



Pivoted double block brake

$$T_B = f.(F_{n1} + F_{n2}).h = 2f.F_n.h$$

Short and Long Shoe Analysis

- In reality constant or uniform constant pressure may not prevail at all points of contact on the shoe.
- • In such case the following general procedure of analysis can be adopted

DRUM BRAKES

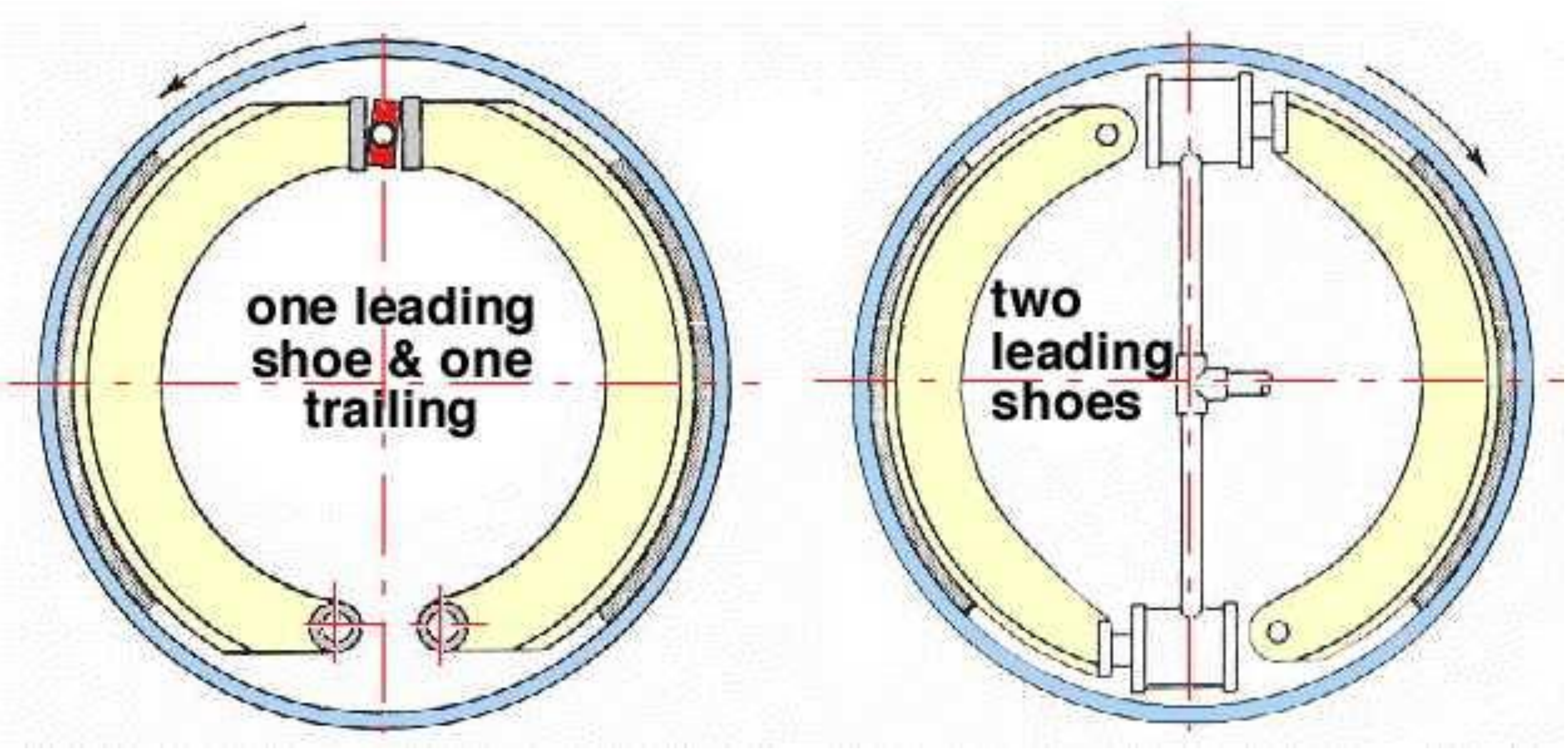
Drum brakes" of the following types are mainly used in automotive vehicles and cranes and elevators.

- Rim types with internal expanding shoes
- Rim types with external contracting shoes

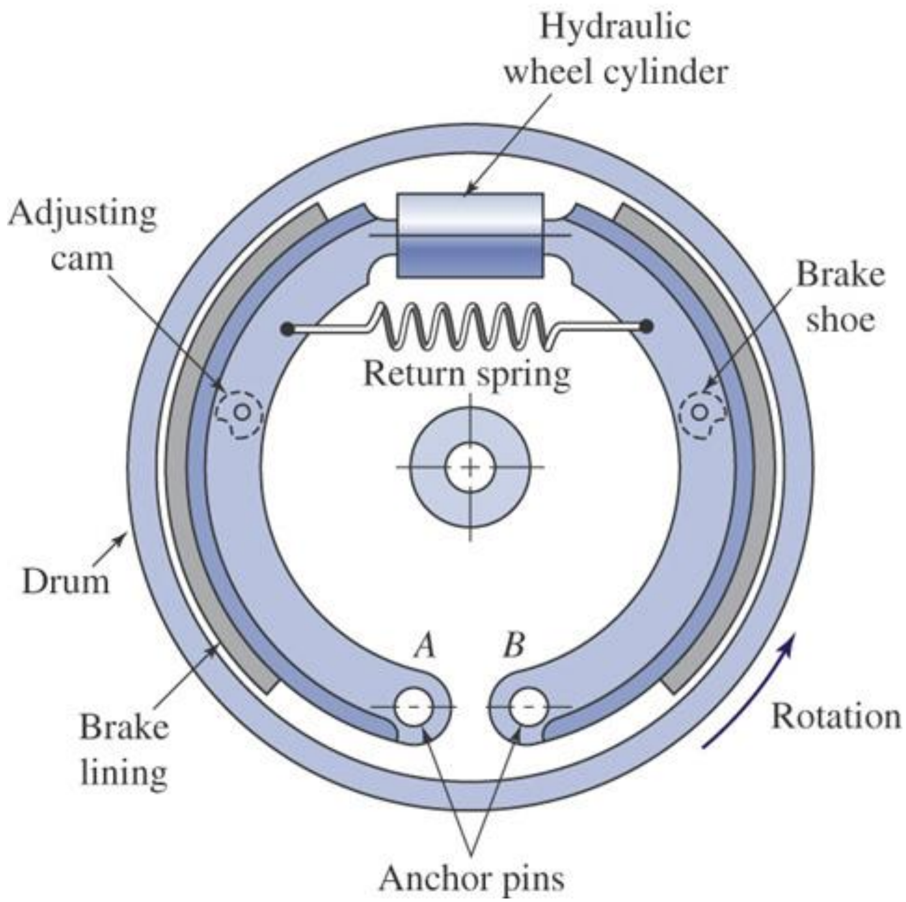
Internal expanding Shoe

- The rim type internal expanding shoe is widely used for braking systems in automotive applications and is generally referred as internal shoe drum brake. The basic approach applied for its analysis is known as long-rigid shoe brake analysis.

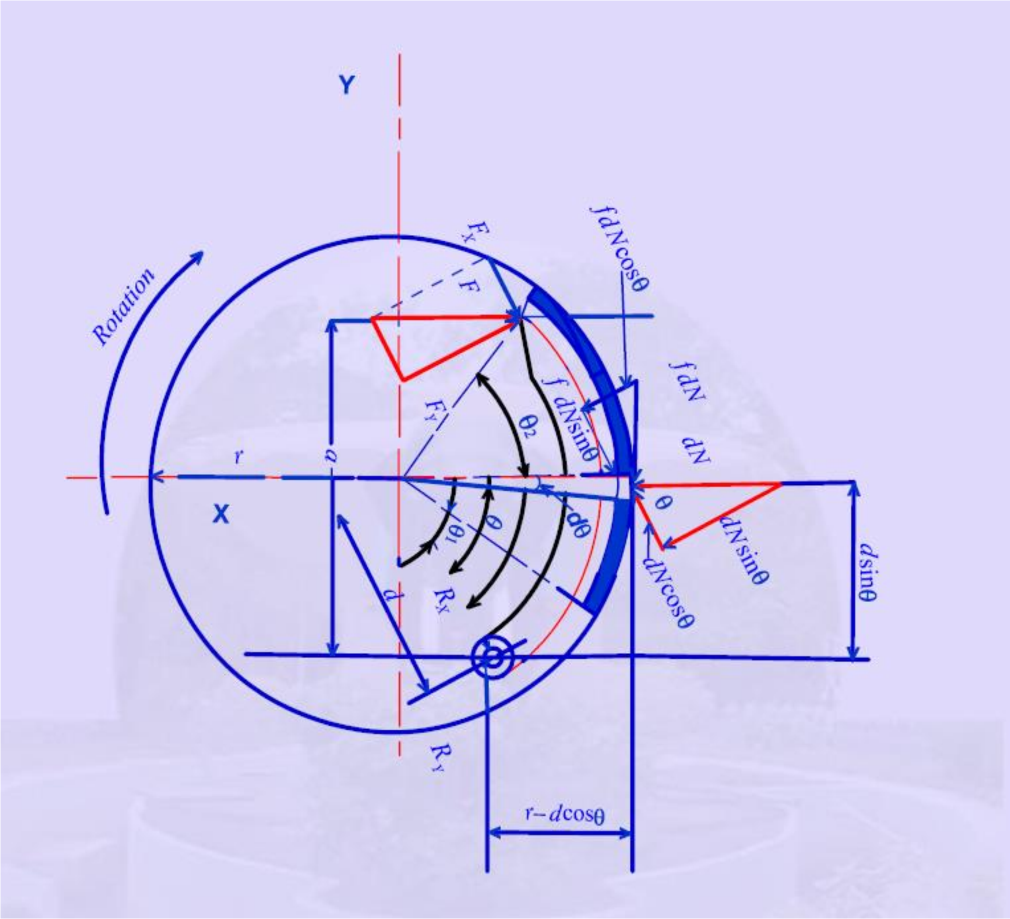
Internal expanding Shoe



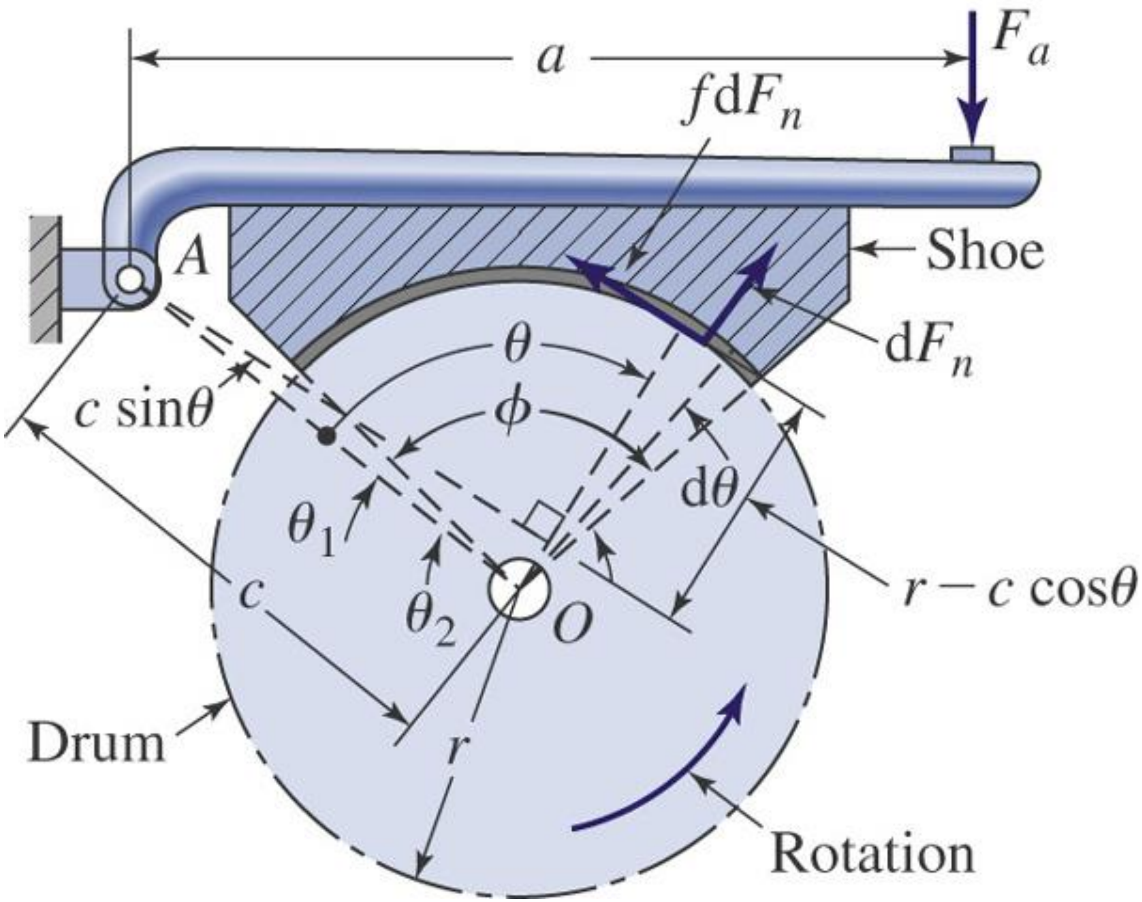
Internal Long-Shoe Drum Brakes



Internal Long-Shoe Drum Brakes



Long-Shoe Drum Brakes



- In this analysis, the pressure at any point is assumed to be proportional to the vertical distance from the hinge pin, the vertical distance from the hinge pin, which in this case is proportional to sine of the angle and thus,

- Since the distance d is constant, the normal pressure at any point is just proportional to sinΘ. Call this constant of proportionality as K

$$p \propto d \sin\theta \propto \sin\theta$$

Thus $p = K \sin\theta$

It the maximum allowable pressure for the lining material is p_{max} *then the constant K can be defined as*

$$K = \frac{p}{\sin\theta} = \frac{p_{max}}{\sin\theta_{max}}$$

$$p = \frac{p_{max}}{\sin\theta_{max}} \sin\theta$$

- The actuating force F is determined by the summation of the moments of normal and frictional forces about the hinge pin and equating it to zero.
- *Summing the moment about point O gives*

$$F = \frac{M_N \pm M_f}{c}$$

where,

- M_n and M_f are the moment of the normal and frictional forces respectively, about the shoe pivot point.
- The sign depends upon the direction of drum rotation,
- - sign for self energizing and + sign for non self energizing

$$M_N = \int_{\theta_2}^{\theta_1} p.b.r.d\theta.d\sin\theta = \int_{\theta_2}^{\theta_1} b.p.r.d.\sin\theta.d\theta$$

$$= \int_{\theta_2}^{\theta_1} b.d.r.\frac{p_{max}}{\sin\theta}\sin^2\theta d\theta$$

$$M_N = \frac{p_{max}b.d.r}{\sin\theta_a}\left[\frac{1}{2}\left(\theta_2 - \theta_1\right) - \frac{1}{4}(\sin 2\theta_2 - \sin 2\theta_1)\right]$$

- *the moment of friction force*

$$M_F = \int_{\theta_2}^{\theta_1} f.p.b.r.d\theta\left(r - d\sin\theta\right)$$

$$= \int_{\theta_2}^{\theta_1} f.b.r.\frac{p_{max}}{\sin\theta_{max}}\sin\theta\left(r - d\sin\theta\right)d\theta$$

$$M_f = \frac{f.p_{max}.b.r}{\sin\theta_a}\left[-r\left(\cos\theta_2 - \cos\theta_1\right) - \frac{d}{2}\left(\sin^2\theta_2 - \sin^2\theta_1\right)\right]$$

- The braking torque T on the drum by the shoe is

$$T = \int_{\theta_1}^{\theta_1} f.b.p.r.d\theta.r$$

$$= \int_{\theta_1}^{\theta_1} fbr^2 \frac{p_{max}}{\sin\theta_{max}} \sin\theta d\theta$$

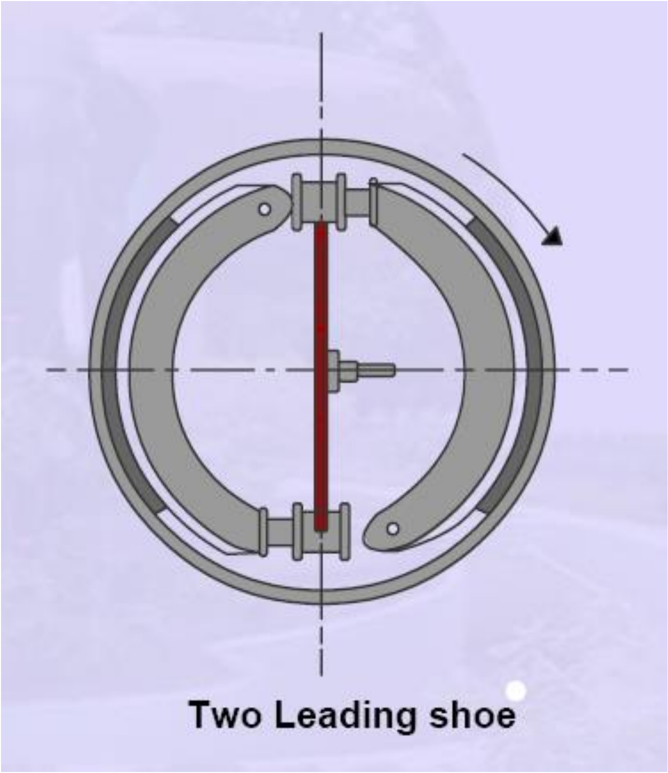
$$T = \frac{fdp_a r^2 (\cos\theta_1 - \cos\theta_2)}{\sin\theta_a}$$

Double Shoe Brakes

- Two Shoes are used to cover maximum area and to minimize the unbalanced forces on the drum, shaft and bearings.

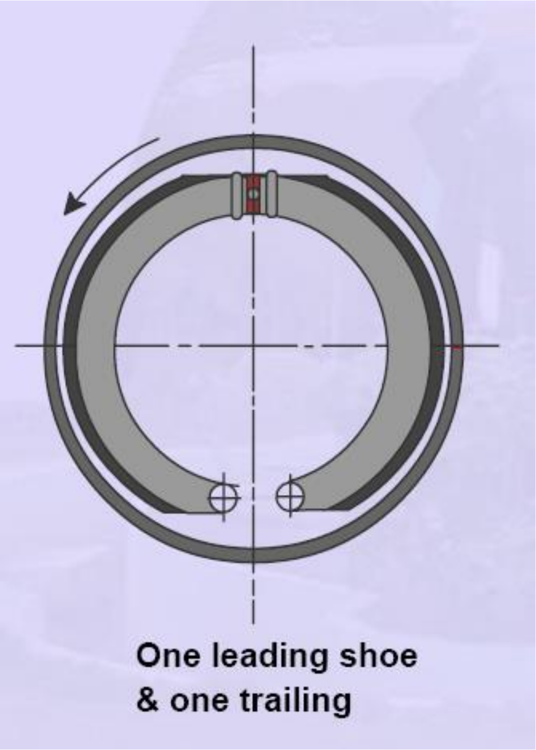
Double Shoe Brakes

- *If both the shoes are arranged such that both are leading shoes in which self energizing are prevailing, then all the other parameters will remain same and the total braking torque on the drum will be twice the value obtained in the analysis.*



Double Shoe Brakes

- *However in most practical applications the shoes are arranged such that one will be leading and the other will be trailing for a given direction of drum rotation*
- *• If the direction of drum rotation changes then the leading shoe will become trailing and vice versa.*



Double Shoe Brakes-
one leading &one trailing

- *Thus this type of arrangement will be equally effective for either direction of drum rotation*
- *Further the shoes can be operated upon using a single cam or hydraulic cylinder thus provide for ease of operation*

Double Shoe Brakes- one leading &one trailing

- However the total braking torque will not be the twice the value of a single shoe,
- This is because the effective contact pressure (force) on the trailing shoe will not be the same, as the moment of the friction force opposes the normal force, there by reducing its actual value as in most applications the same normal force is applied or created at the point of force application on the brake shoe as noted above

Double Shoe Brakes-
one leading &one trailing

- *Consequently we may write the actual or effective pressure prevailing on a trailing shoe*

$$p_a' = p_a \cdot \left[\frac{F.a}{(M_n + M_f)} \right]$$

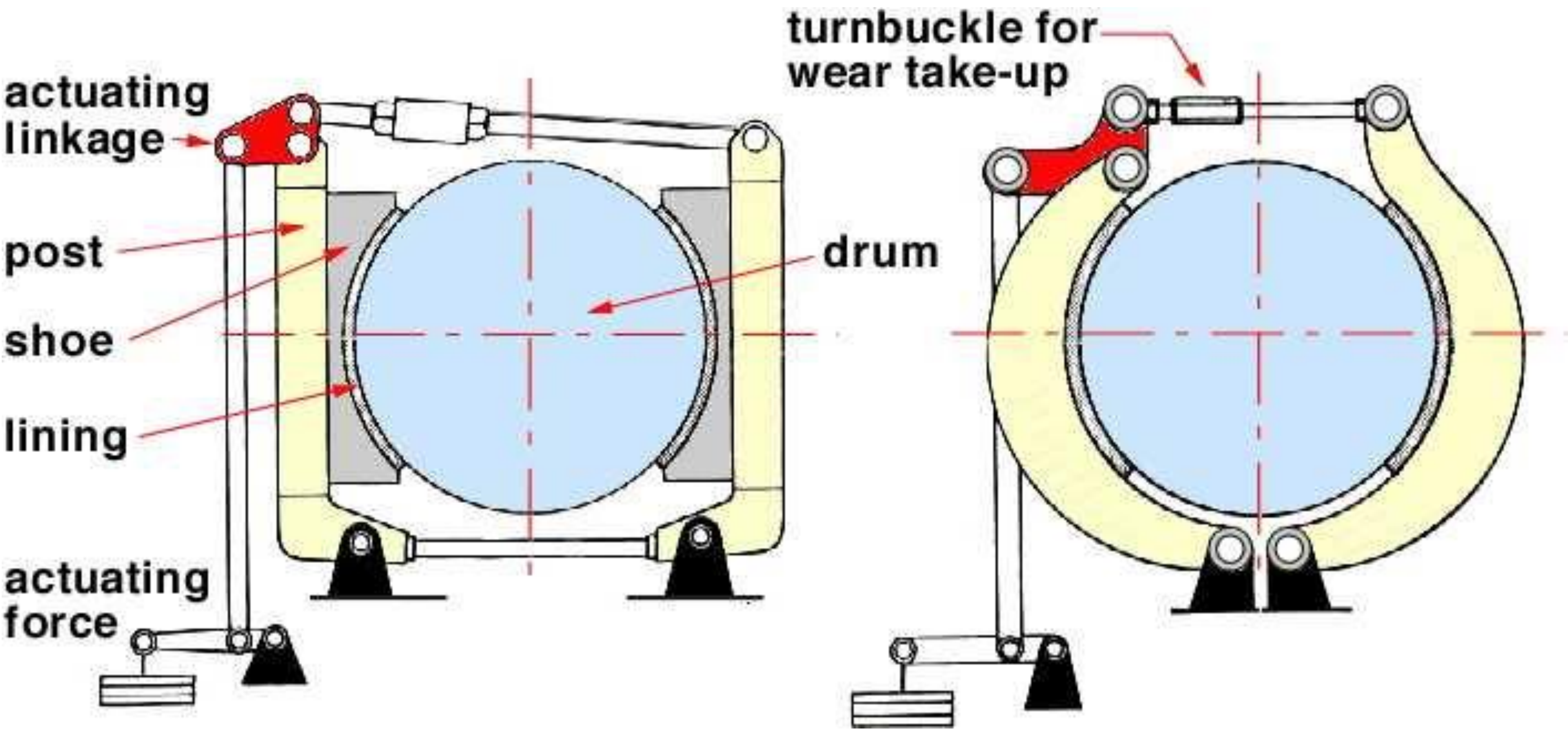
Resulting equation for the braking torque

$$T_B = f.w.r^2 . \frac{p_a}{\sin \theta_a} (\cos \theta_1 - \cos \theta_2)(p_a + p_a')$$

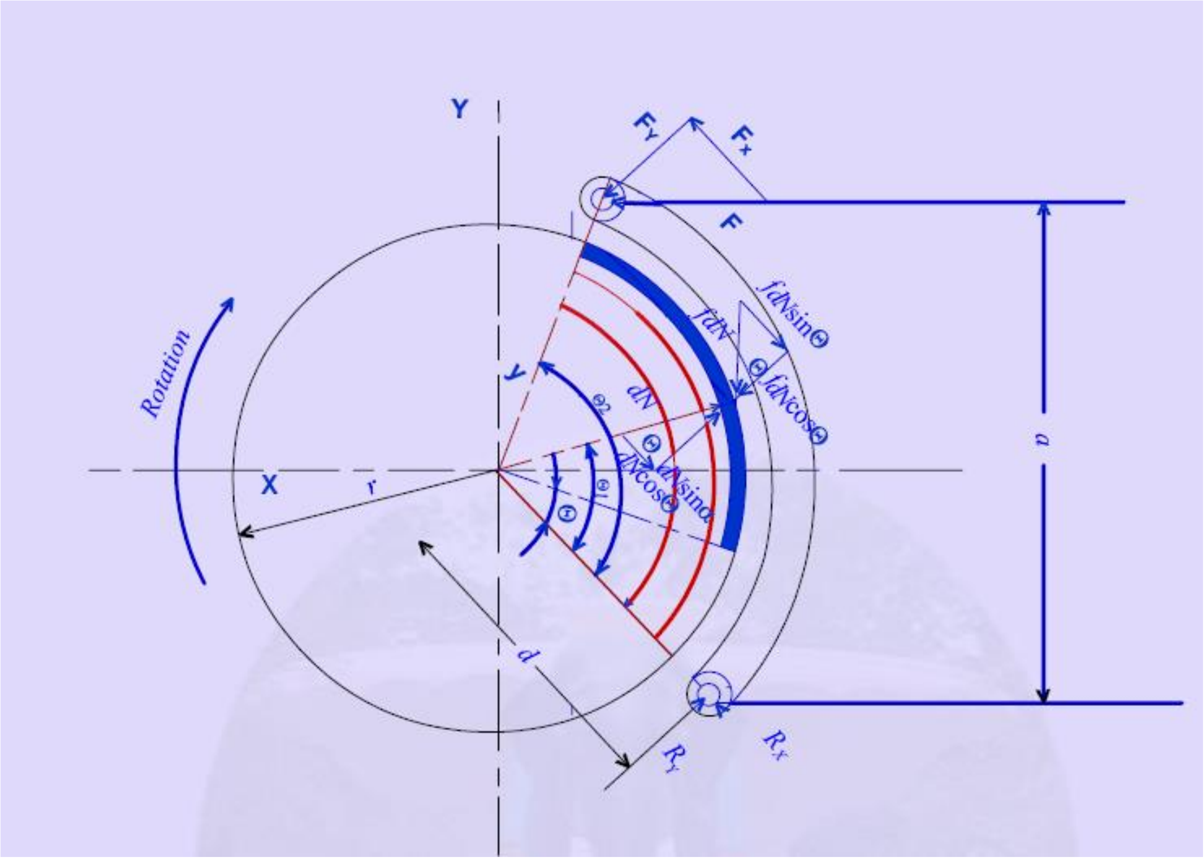
External Contracting Rigid Shoes

- These are ***external rigid shoe*** brakes - **rigid** because the shoes with attached linings are rigidly connected to the pivoted posts; **external** because they lie outside the rotating drum. An actuation linkage distributes the actuation force to the posts thereby causing them both to rotate towards the drum - the linings thus contract around the drum and develop a friction braking torque.

External Contracting Rigid Shoes



External Contracting Rigid Shoes



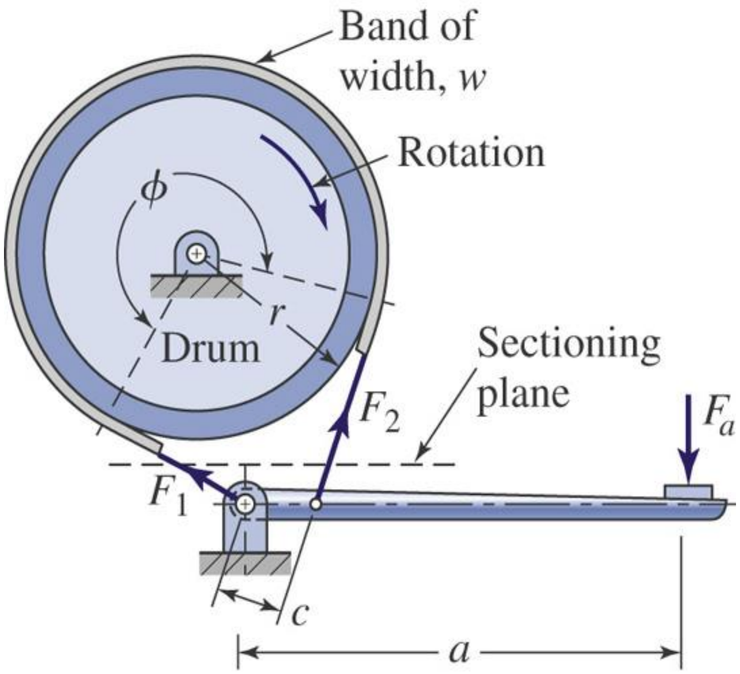
External Contracting Rigid Shoes

- The resulting equations for moment of normal and frictional force as well as the actuating force and braking torque are same as seen earlier.
- As noted earlier for the internal expanding shoes, for the double shoe brake the braking torque for one leading and one trailing shoe acted upon a common cam or actuating force the torque equation developed earlier can be applied

Band Brakes

- A ***band brake*** consists of a flexible band faced with friction material bearing on the periphery of a drum which may rotate in either direction.
- The ***actuation force*** P is applied to the band's extremities through an ***actuation linkage*** such as the cranked lever illustrated. Tension build-up in the band is identical to that in a stationary flat belt.

Simple Band Brake



$T = (F1 – F2)r$

Simple Band Brake

- For the cw rotation of the drum the tensions in the tight side (F_1) and slack side (F_2) are related by

  $\mathbf{F_1/F_2 = e^{f\theta}}$,

  where

  f = coefficient of friction

  θ = Angle of contact

Simple Band Brake

For the simple band brake arrangement as shown above, the actuating force F_a is given as,

$$\mathbf{F_a = c\, F_2 / a}$$

If the direction of rotation is changed to ccw, then the tight and slack sides are reversed. Therefore the actuating force is

$$\mathbf{F_a = c\, F_1 / a}$$

Simple Band Brake

Since the tension in the band is not a constant, the pressure is a maximum at the tight side and a minimum at the slack side, so

$$\mathbf{p_{max} = F_1 / w\ r\ ,}$$

$$\mathbf{p_{min} = F_2 / w\ r,}$$

where w= width of the band

Simple Band Brake

- The band thickness is given empirically as

$h = 0.005D$, D=drum diameter

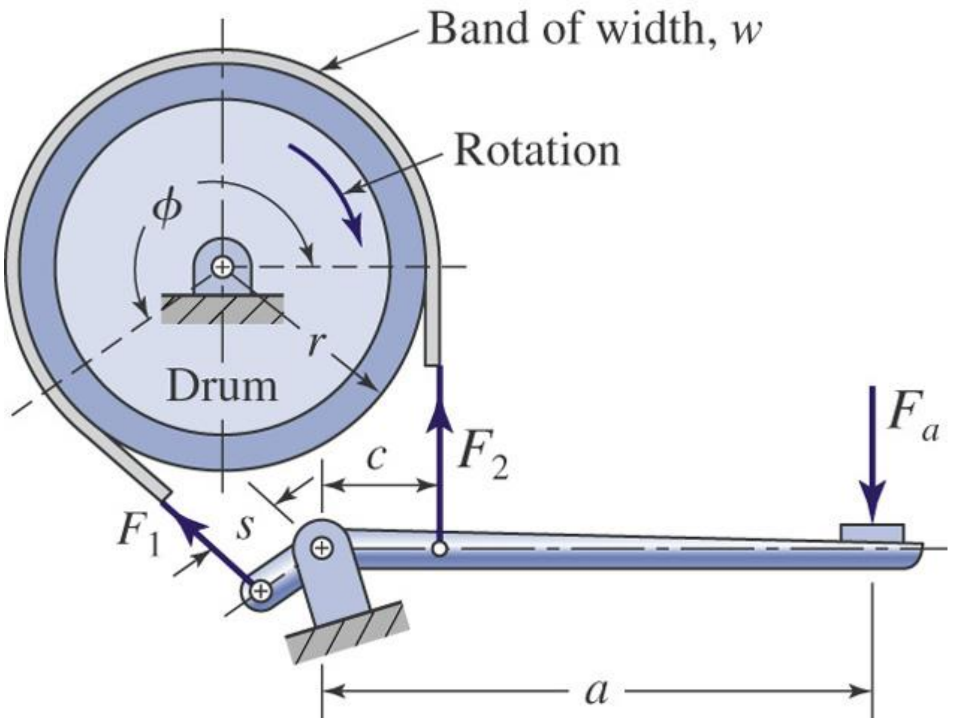
- The width of the band **$w = F_1 / h\,\sigma_d$,**

, σ_d = design stress of the band material

Differential Band Brake

- For the differential band brakes, neither ends of the band are connected to the fulcrum.
- The differential band brake as shown below can be configured to be self energizing and can be arranged to operate in either direction.
- The friction force is F_1-F_2 and it acts in the direction of F_1, and therefore a band brake will be self energizing when F_1 acts to apply the brake as shown below.

Differential Band Brake



Differential Band Brake

- The equation for the actuating force is obtained by summing the moments about the pivot point. Thus,

$F_a\, a + F_1\, s - F_2\, c = 0$

$F_a = (F_2\, c - F_1\, s) / a$

- If $F_2\, c \leq F_1\, s$, the actuating force will be negative or zero as the case may be and the brake is called as self-locking.
- It is also apparent that the brake is effectively free wheeling in the opposite direction. The differential brake can therefore be arranged to enable rotation in one direction only.

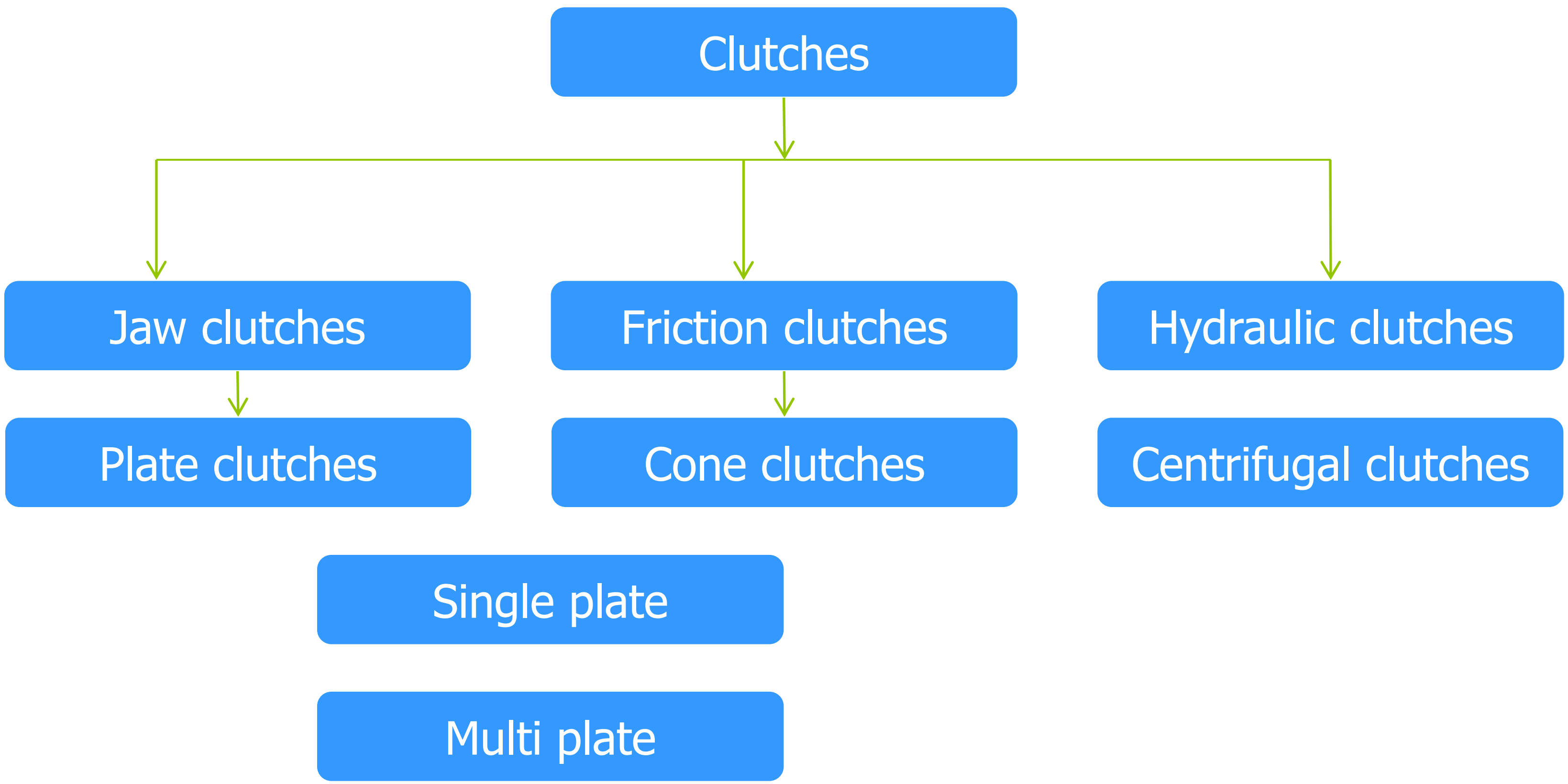
Chapter#14 Clutches

What is the purpose of the Clutch?

Purpose of the Clutch

- Allows engine to be *disengaged* from transmission for shifting gears and coming to a stop
- Allows smooth *engagement* of engine to transmission

Clutch classification



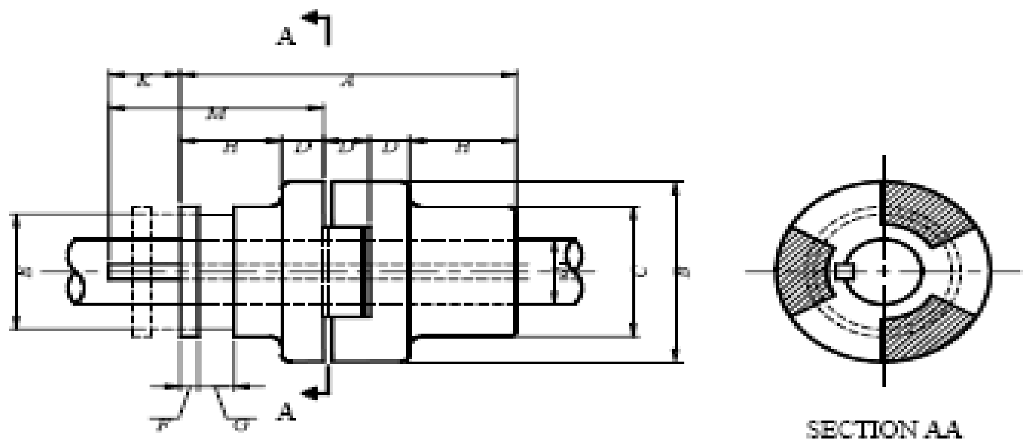
Jaw Clutches

- A clutch that consists of two mating surfaces with interconnecting elements, such as teeth, that lock together during engagement to prevent slipping.

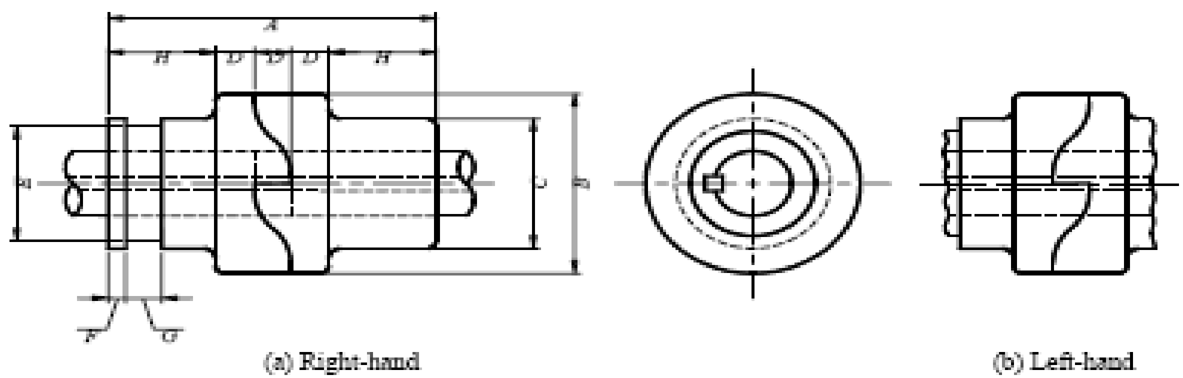
- These clutches are used to positively connect and disconnect rotating shafts where engagement and disengagement is not frequent.

- Square jaws are used where engagement and disengagement under power or in motion is not required. They transmit power in both directions.

- Spiral jaw are made in right or left hand style and will transmit power in only one direction. They can be engaged or disengaged at low speeds



A. Square jaw clutch



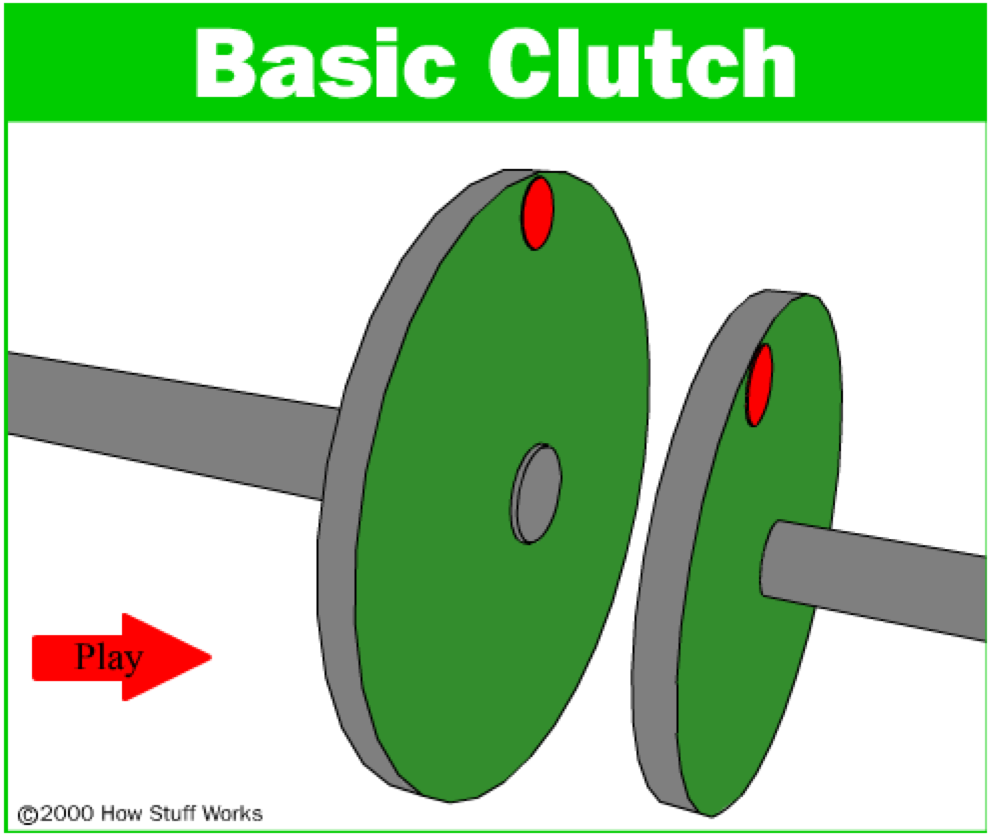
B. Spiral jaw clutch

Jaw Clutches

Friction clutches

- A mechanical clutch that transmits torque through surface friction between the faces of the clutch
- Friction clutches have pairs of conical , disk, or ring-shaped mating surfaces and means for pressing the surfaces together. The pressure may be created by a spring or a series of levers locked in position by the wedging action of a conical spool.

Friction clutches



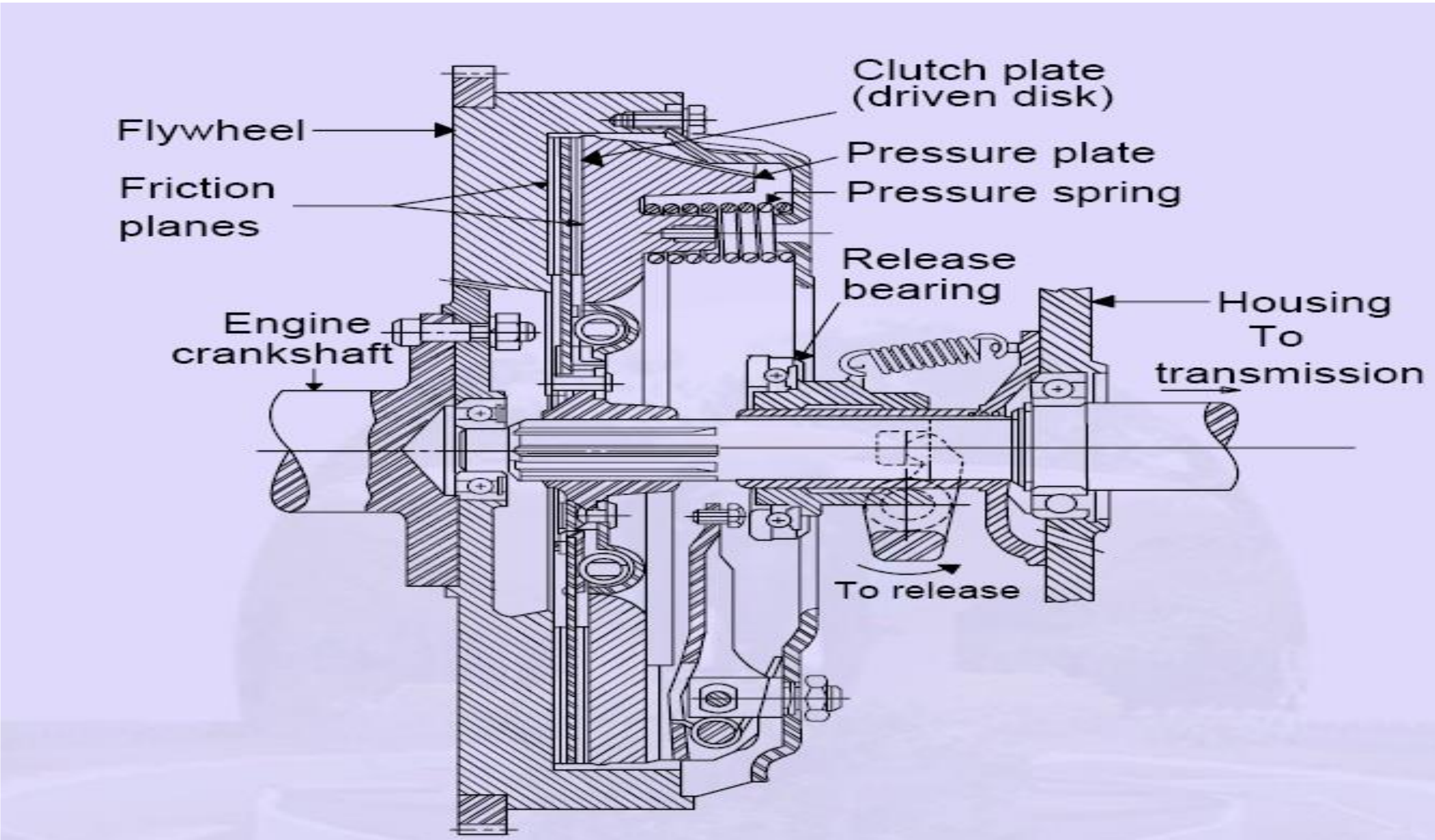
Friction clutches-Wet and dry clutches

- A 'wet clutch' is immersed in a cooling lubricating fluid, which also keeps the surfaces clean and gives smoother performance and longer life. Wet clutches, however, tend to lose some energy to the liquid. A 'dry clutch', as the name implies, is not bathed in fluid. Since the surfaces of a wet clutch can be slippery (as with a motorcycle clutch bathed in engine oil), stacking multiple clutch disks can compensate for slippage.

Clutch construction

- Basically, the clutch needs three parts. These are the engine flywheel, a friction disc called the clutch plate and a pressure plate. When the engine is running and the flywheel is rotating, the pressure plate also rotates as the pressure plate is attached to the flywheel. The friction disc is located between the two

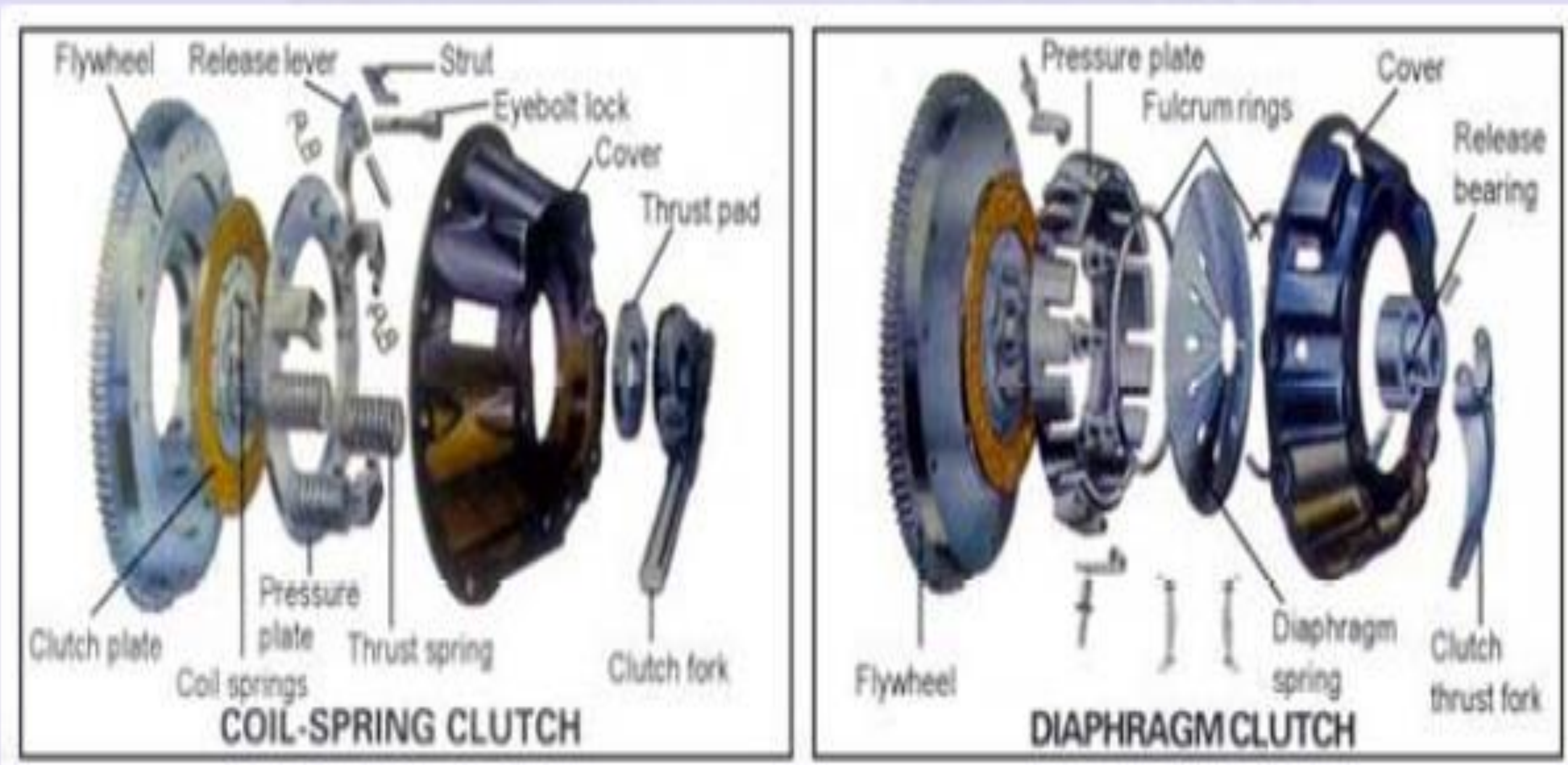
Clutch construction



Clutch construction

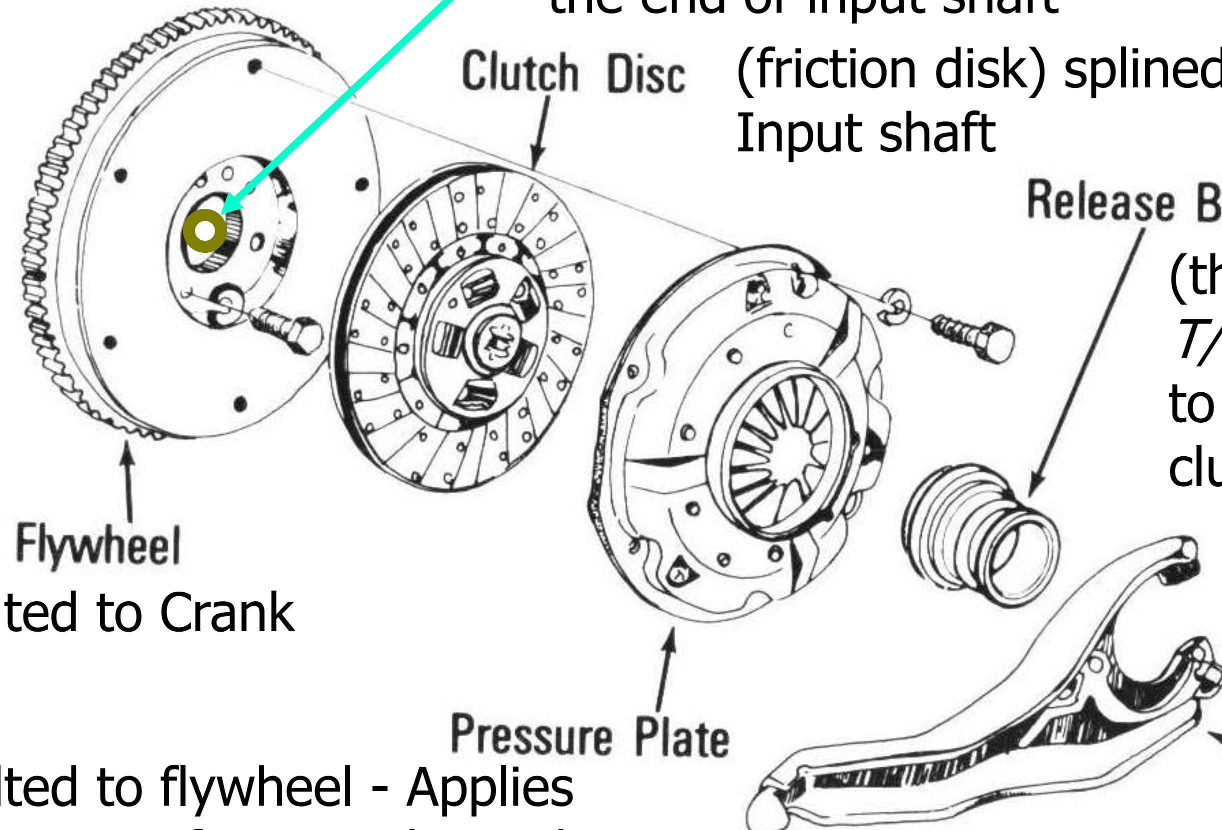
Two basic types of clutch are the coil-spring clutch and the diaphragm-spring clutch. The difference between them is in the type of spring used. The coil spring clutch uses coil springs as pressure springs pressure. The diaphragm clutch uses a diaphragm spring.

Clutch construction



Clutch construction

Pilot bushing or bearing in canter of flywheel or crankshaft, supports the end of input shaft



Clutch Disc (friction disk) splined to transmission Input shaft

Release Bearing (throw-out bearing *T/O bearing*) allows to push on rotating clutch fingers

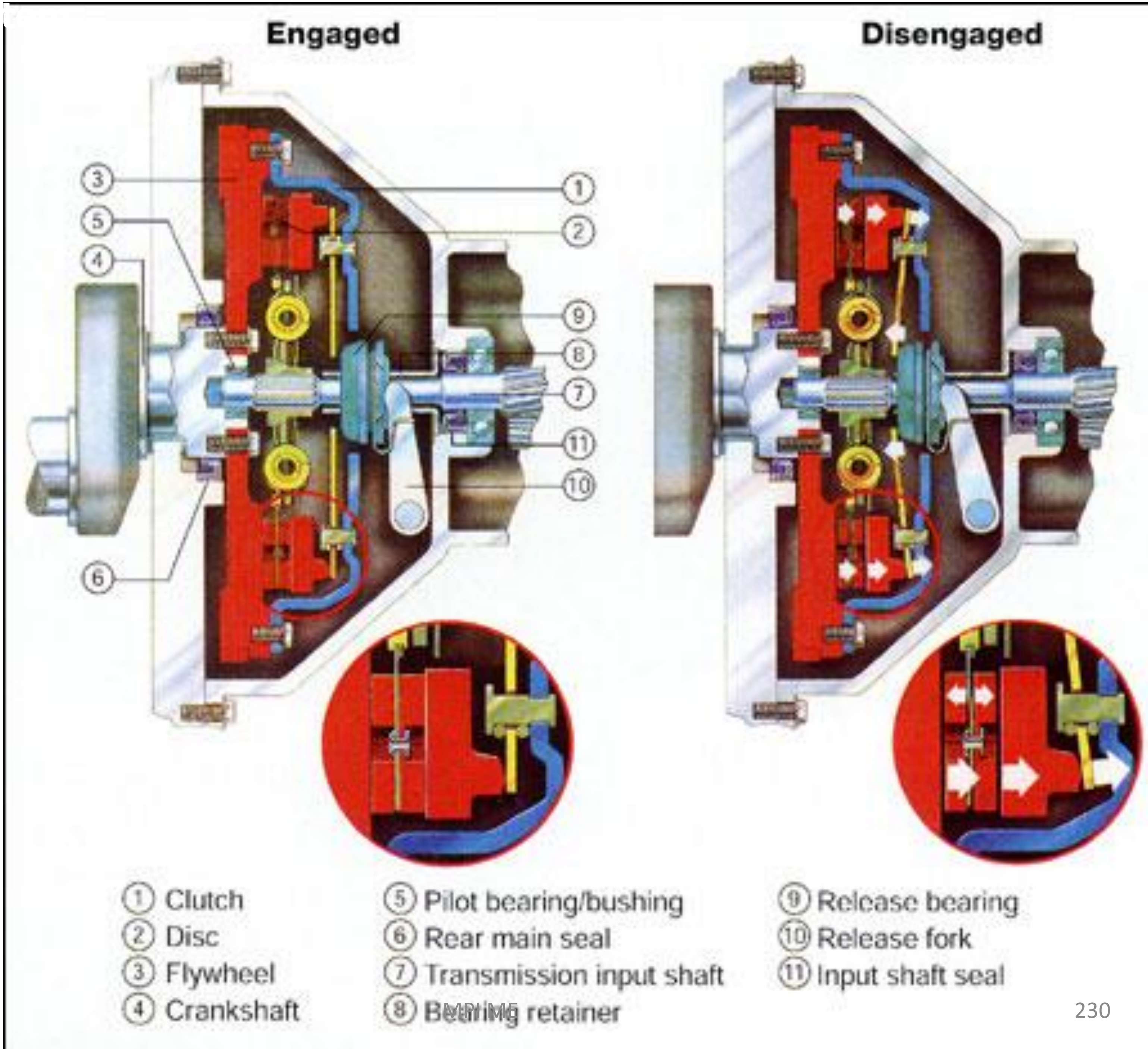
Flywheel Bolted to Crank

Pressure Plate Bolted to flywheel - Applies the spring force to clamp the friction disk to the flywheel

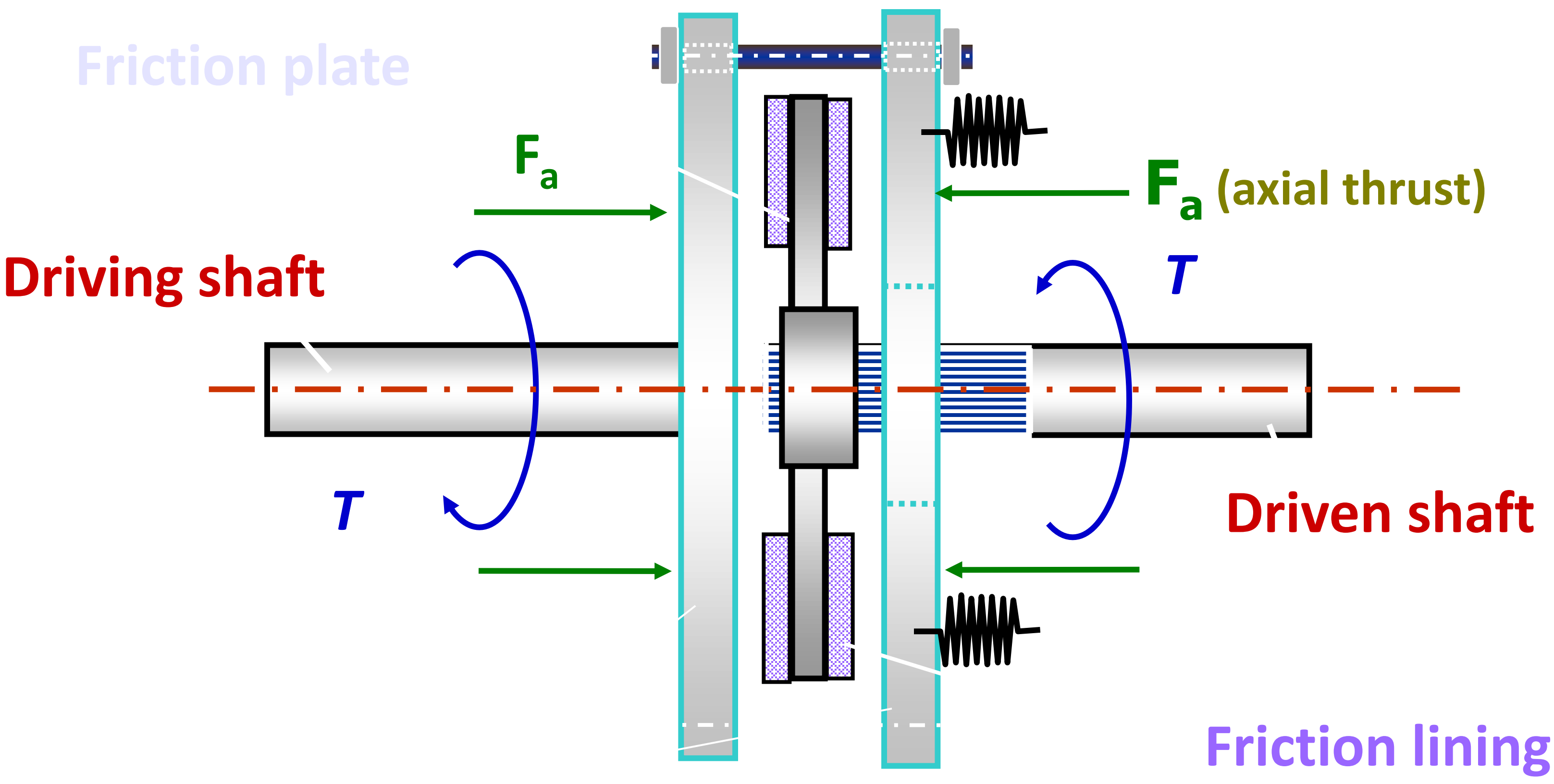
Release Fork (clutch fork) pushes T/O bearing to release rotating clutch

Clutch operation

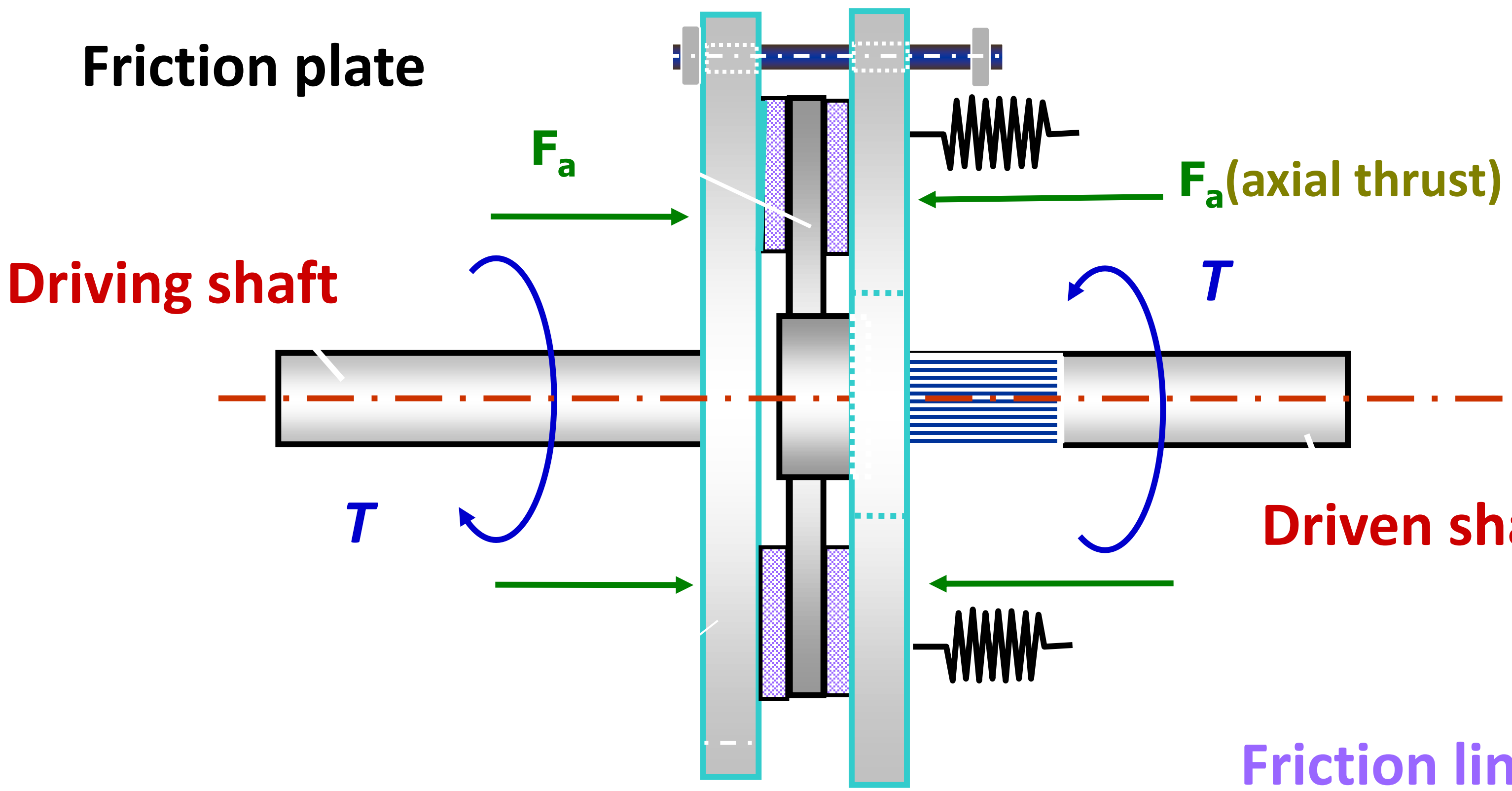
- When the driver has pushed down the clutch pedal the clutch is released. This action forces the pressure plate to move away from the friction disc. There are now air gaps between the flywheel and the friction disc, and between the friction disc and the pressure plate. No power can be transmitted through the clutch
- When the driver releases the clutch pedal, power can flow through the clutch. Springs in the clutch force the pressure plate against the friction disc. This action clamps the friction disk tightly between the flywheel and the pressure plate. Now, the pressure plate and friction disc rotate with the flywheel.

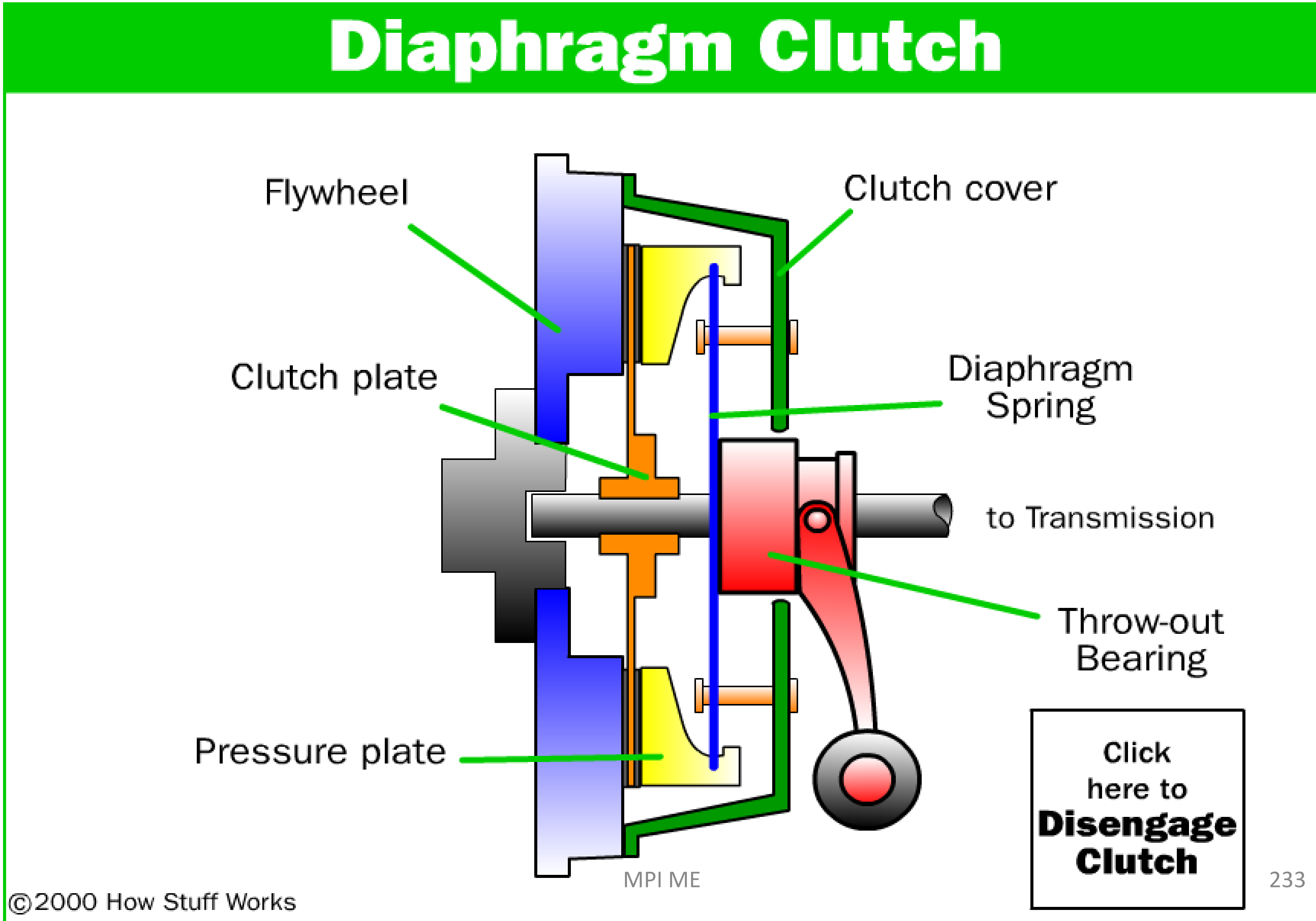

Engaged
Disengaged
① Clutch
② Disc
③ Flywheel
④ Crankshaft
⑤ Pilot bearing/bushing
⑥ Rear main seal
⑦ Transmission input shaft
⑧ Bearing retainer
⑨ Release bearing
⑩ Release fork
⑪ Input shaft seal

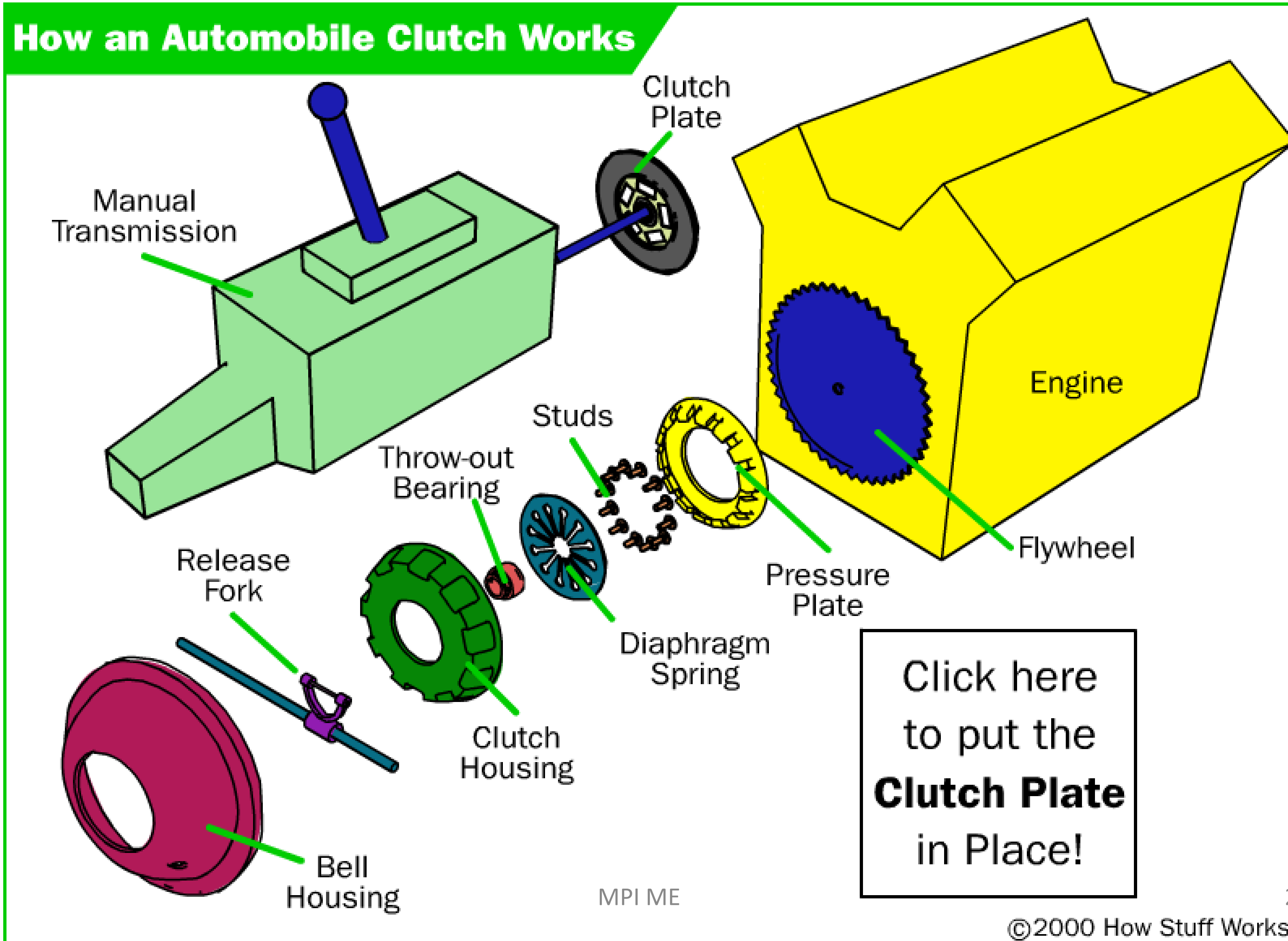
Single-plate Friction Clutch (Disengaged position)



Single-plate Friction Clutch (Engaged position)




Diaphragm Clutch
Flywheel
Clutch cover
Clutch plate
Diaphragm Spring
to Transmission
Throw-out Bearing
Pressure plate
Click here to Disengage Clutch
©2000 How Stuff Works


How an Automobile Clutch Works
Clutch Plate
Manual Transmission
Engine
Studs
Throw-out Bearing
Flywheel
Release Fork
Pressure Plate
Diaphragm Spring
Clutch Housing
Bell Housing
Click here to put the **Clutch Plate** in Place!
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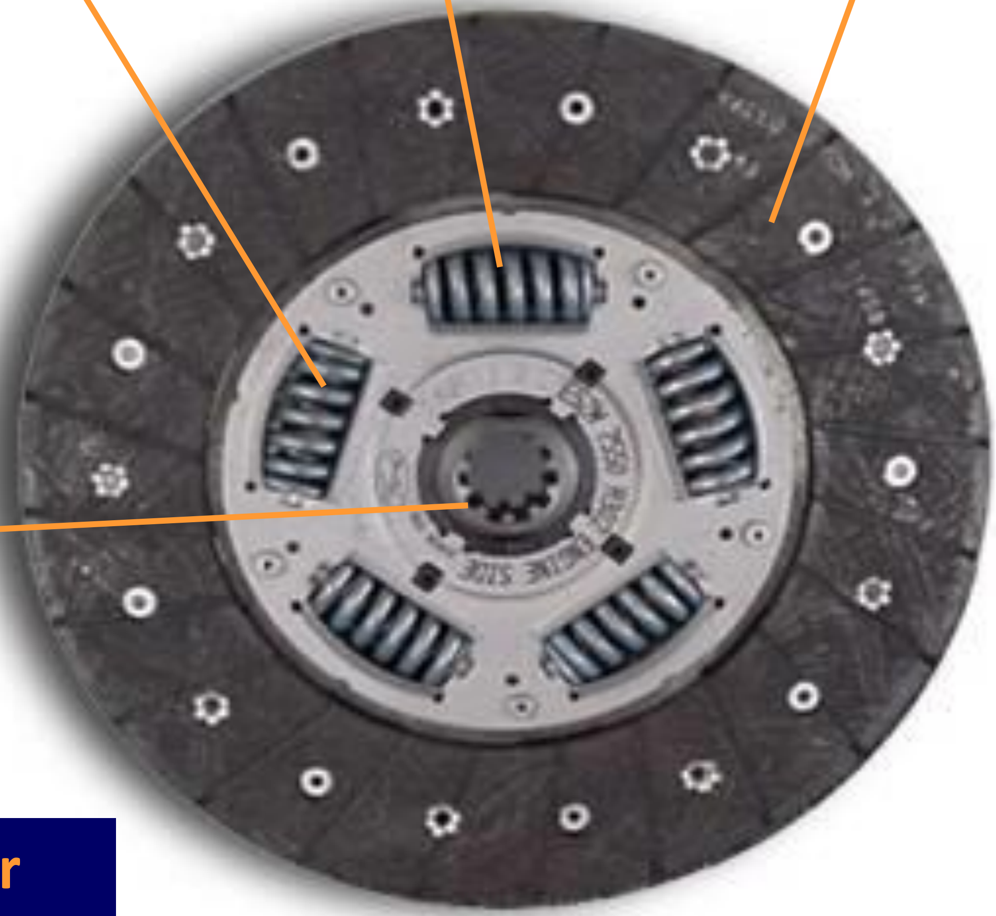
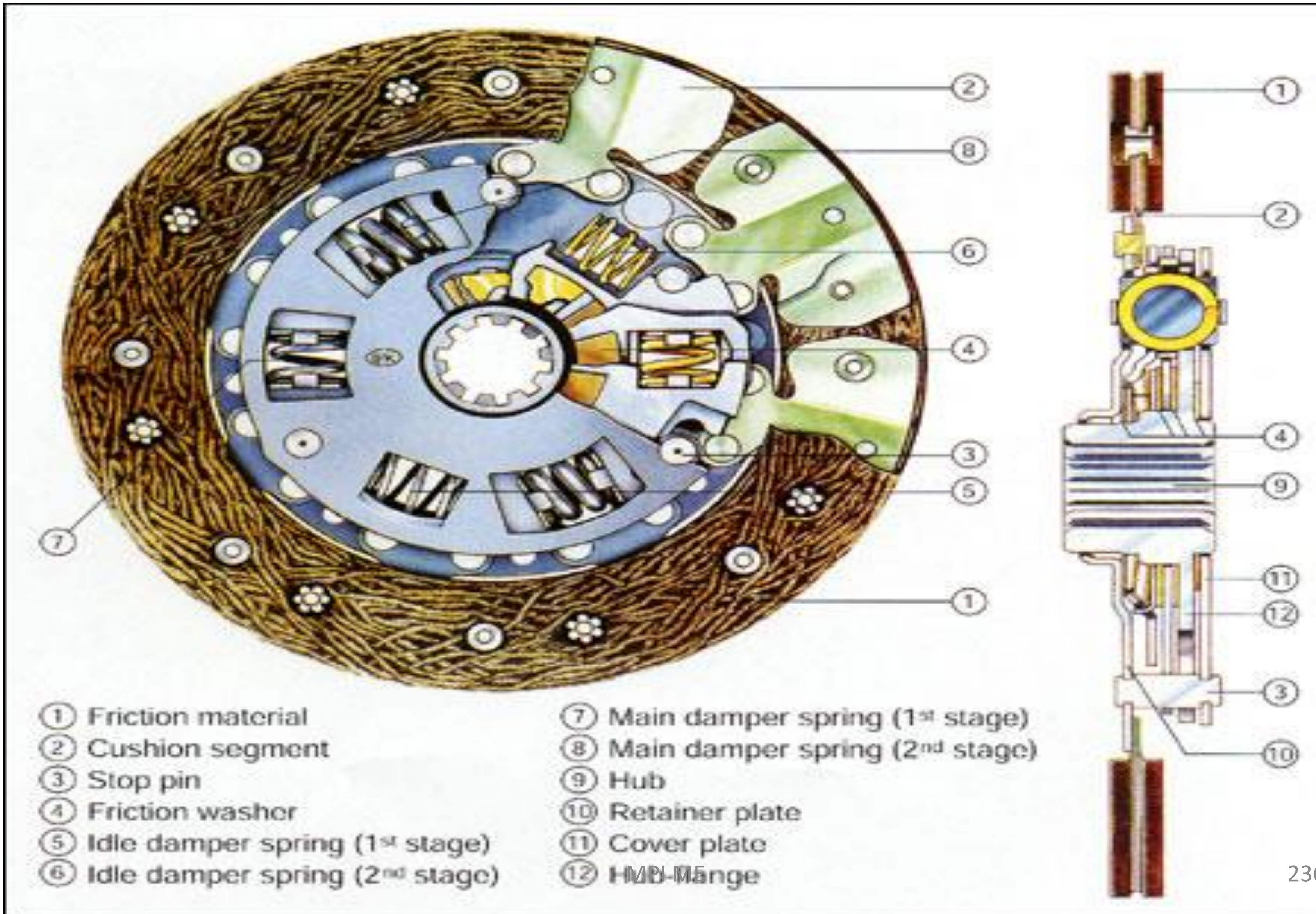


Clutch Plate or Friction Plate

Clutch Plate or Friction Plate



Clutch disc

- Friction material disc splined to input shaft
- Friction material may contain ASBESTOS
- Friction material can be bonded or riveted
- Friction is attached to wave springs
- Most have torsional dampener springs
- Normal wearing component
- Normally a worn out disc will cause slipping

Friction lining materials

Typical characteristics of some widely used friction linings are given in the table

Table Properties of common clutch/ Brake lining materials

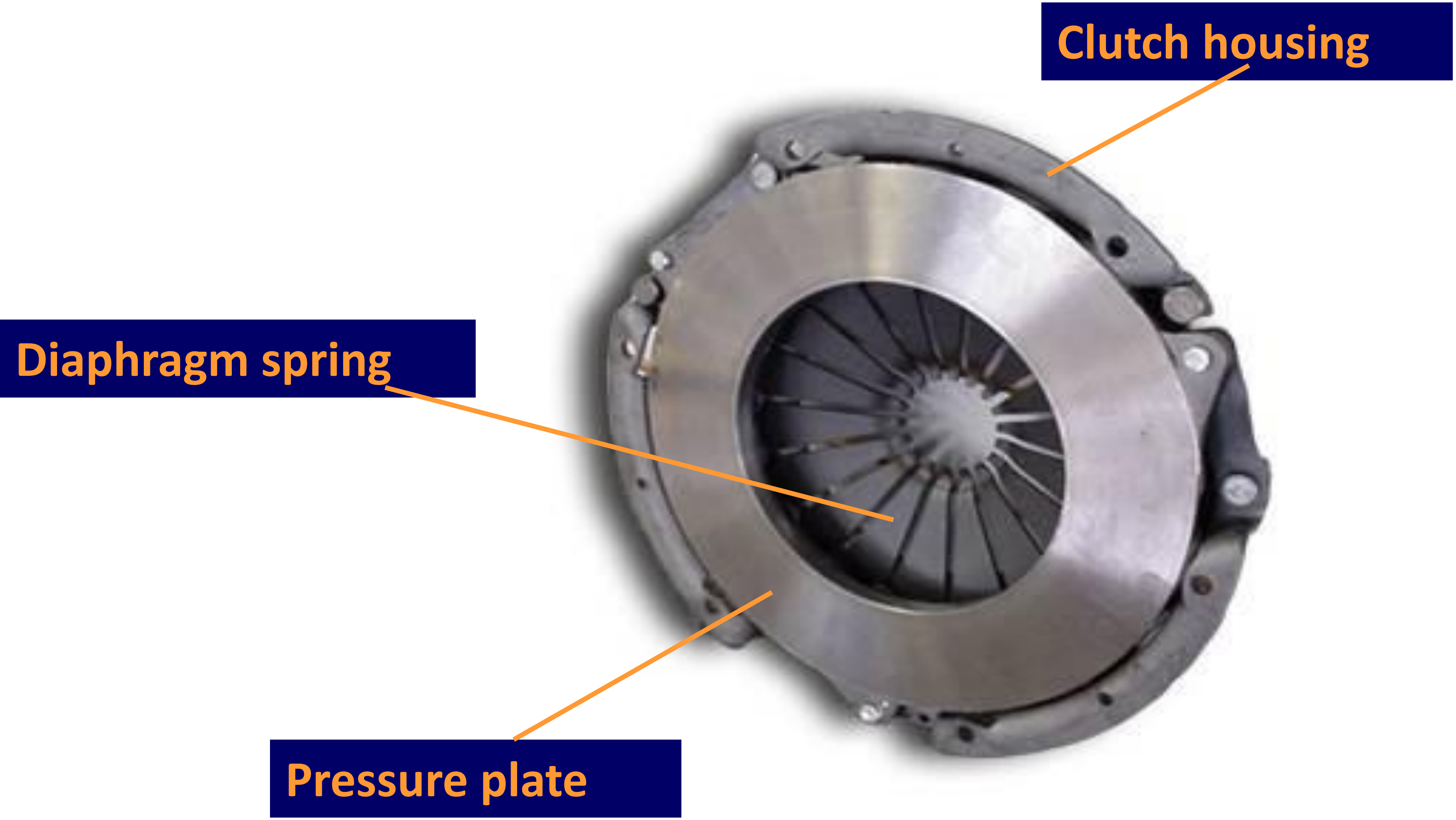
Friction Material Against Steel or CI	Dynamic Coefficient of Friction		Maximum Pressure KPa	Maximum Temprerature °C
	dry	in oil		
Molded	0.25-0.45	0.06-0.09	1030-2070	204-260
Woven	0.25-0.45	0.08-0.10	345-690	204-260
Sintered metal	0.15-0.45	0.05-0.08	1030-2070	232-677
Cast iron of hard steel	0.15-0.25	0.03-0.06	690-720	260

For wet condition

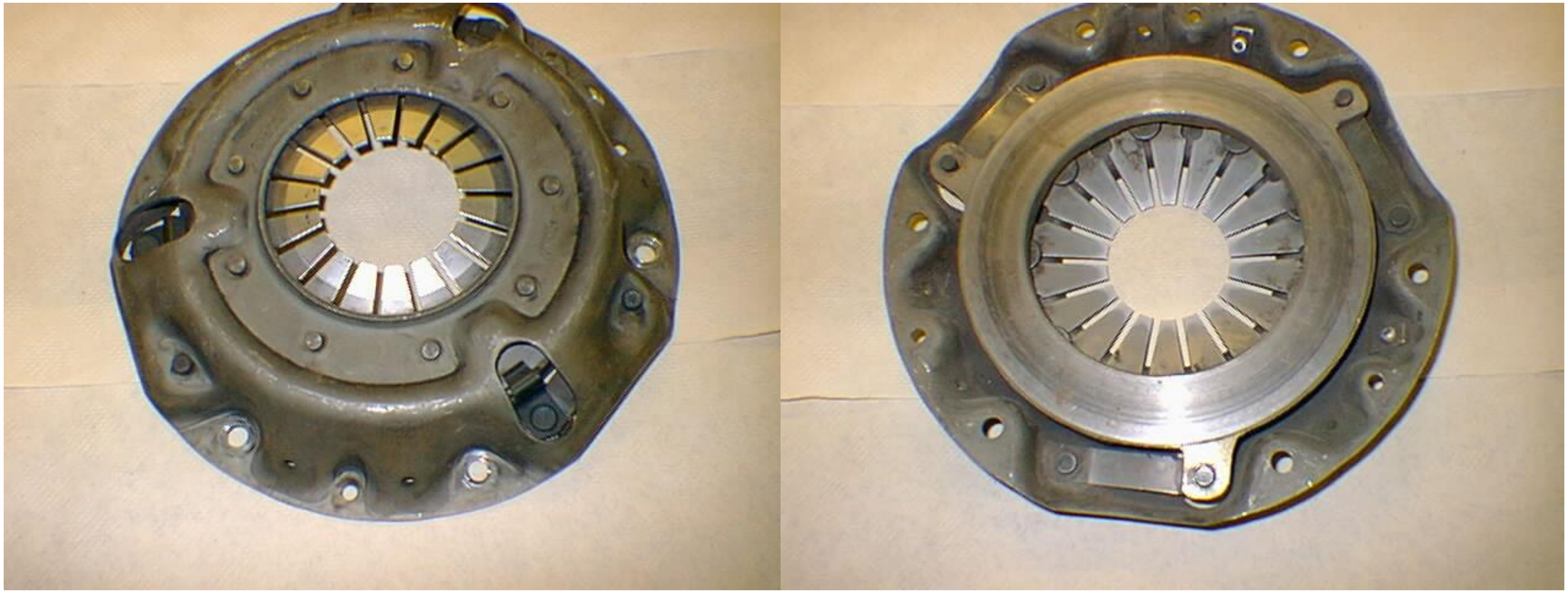
Friction Material[a]	Dynamic Friction Coefficient f
Molded	0.06–0.09
Woven	0.08–0.10
Sintered metal	0.05–0.08
Paper	0.10–0.14
Graphitic	0.12 (avg.)
Polymeric	0.11 (avg.)
Cork	0.15–0.25
Wood	0.12–0.16
Cast iron, hard steel	0.03–0.06

[a]When rubbing against smooth cast iron or steel.

Clutches



Pressure plate



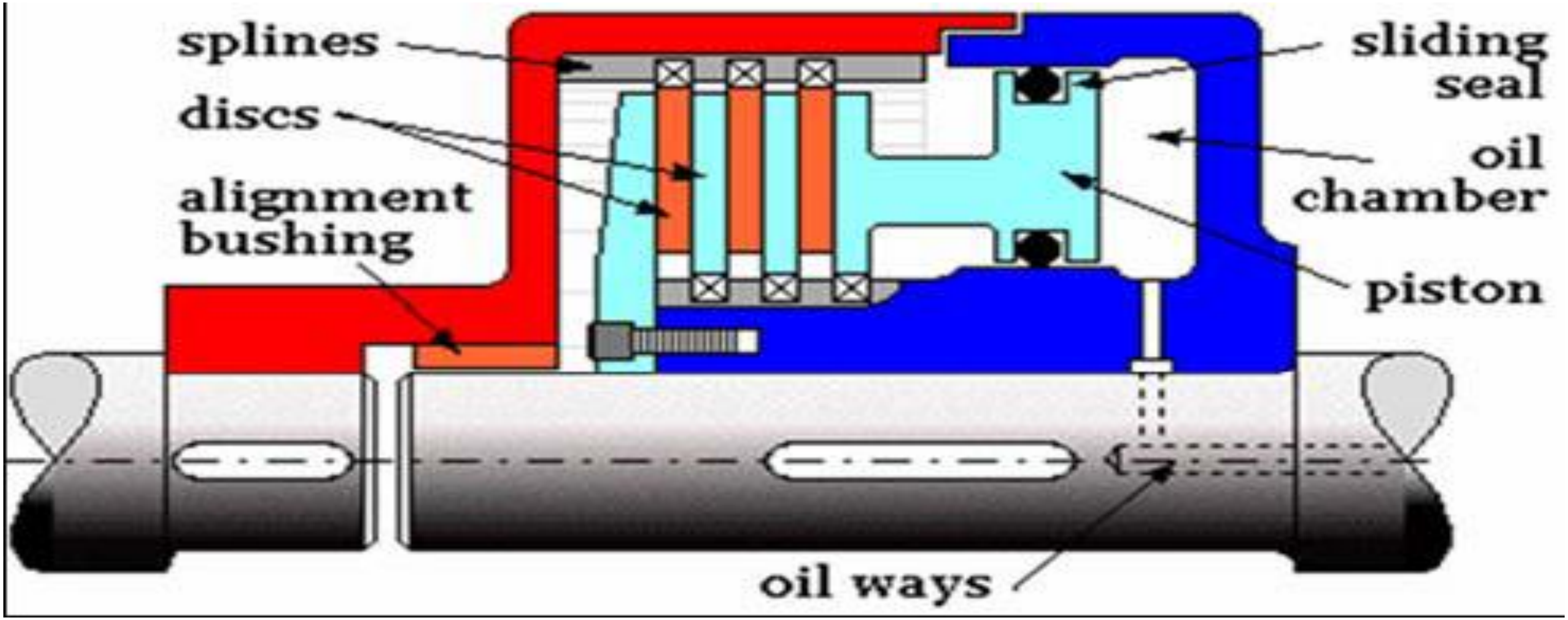
Pressure plate

- Provides clamping pressure to disc
  - Works like spring loaded clamp
- Bolted to flywheel
- Can use Belleville spring acted on by Throw Out bearing
- Can use coil springs and levers acted on by Throw Out bearing

Clutch linkage

- Can be operated by cable, rods or hydraulics
- May be automatic or manually adjusted
- Hydraulic will have a master and slave cylinder
  - Will use brake fluid for hydraulic action
  - Will need bleeding with repairs

Multiple plate wet clutch



Design Analysis of Friction clutches

To design analyze the performance of these devices, a knowledge on the following are required.

- *1. The torque transmitted*
- *2. The actuating force.*
- *3. The energy loss*
- *4. The temperature rise*

Method of Analysis

- Uniform Pressure Conditions

The torque that can be transmitted by a clutch is a function of its geometry and the magnitude of the actuating force applied as well as the condition of contact prevailing between the members. The applied force can keep the members together with a uniform pressure all over its contact area and the consequent analysis is based on uniform pressure condition

Method of Analysis

- Uniform Wear Conditions

However as the time progresses some wear takes place between the contacting members and this may alter or vary the contact pressure appropriately and uniform pressure condition may no longer prevail. Hence the analysis here is based on uniform wear condition

Method of Analysis

Assuming uniform pressure, p, the torque transmission capacity, T is given by,

$$T = \int_{r_i}^{r_o} 2\pi p f r^2 \, dr = \frac{2}{3}\pi p f (r_o^3 - r_i^3)$$

Uniform pressure condition

- The actuating force, F_a that need to be applied to transmit this torque is given by,

$$F_a = \int_{r_i}^{r_o} 2\pi p r\,dr$$

$$F_a = \pi\left(r_o^2 - r_i^2\right).p$$

Uniform pressure condition

The above equations can be combined together to give equation for the torque as

$$T = fF_a \cdot \frac{2}{3} \frac{(r_o^3 - r_i^3)}{(r_o^2 - r_i^2)}$$

Uniform pressure condition

- Again, the equation of torque can be written as,

$$T = ½\, f\, F_a\, D_m$$

D_m = mean diameter

$$= 2/3\ [(D_o^3 - D_i^3)/\ (D_o^2 - D_i^2)]$$

Uniform pressure condition

- In a plate clutch, the torque is transmitted by friction between one (single plate clutch) or more (multiple plate clutch) pairs of co-axial annular driving faces maintained in contact by an axial thrust.
- If both sides of the plate are faced with friction material, so that a single-plate clutch has two pairs of driving faces in contact.

Then, for a plate clutch, the maximum torque transmitted is

$$T = \tfrac{1}{2}\, n'\, f\, F_a\, D_m$$

n' = no. of pairs of driving faces

Uniform wear condition

- According to some established theories the wear in a mechanical system is proportional to the ‘pv’ factor where p refers the contact pressure and v the sliding velocity. Based on this for the case of a plate clutch we can state

constant-wear rate R_w =pv =constant

Uniform wear condition

putting, velocity, $v = r\omega$

$$R_w = pr\omega = \text{constant}$$

Assuming a constant angular velocity

$$pr = \text{constant}$$

The largest pressure p_{max} must then occur at the smallest radius r_i

$$P_{max} r_i = \text{constant}$$

Hence pressure at any point in the contact region

$$p = p_{max} \frac{r_i}{r}$$

Uniform wear condition

- The axial force Fa is given by,

$$F = \int_{r_i}^{r_o} 2\pi p r dr = \int_{r_i}^{r_o} 2\pi \left(p_{max} \frac{r_i}{r} \right) r dr = 2\pi p_{max} r_i (r_o - r_i)$$

- The torque transmission capacity is given by,

$$T = \int_{r_i}^{r_o} f 2\pi p_{max} r_i r dr = f\pi p_{max} r_i (r_o^2 - r_i^2)$$

Uniform wear condition

- Combining the two equations, we get

Torque

$$T = fF_a \cdot \frac{(r_o + r_i)}{2}$$

Uniform wear condition

- Again, torque equation can be written as $T = 1/2\ f\ F_a\ D_m$ where,

mean diameter $D_m = (D_o + D_i)/2$.

If the clutch system have n' pairs of friction surfaces, then torque,

$$T = 1/2\ n'\ f\ F_a\ D_m$$

Design Torque

The design torque is given by

$$T_d = \beta T$$

where β is the engagement factor or the friction margin factor. It accounts for any slippage during transmission.

T =Torque to be transmitted

Power transmitted

- The power transmitting capacity of the clutch system is given by,

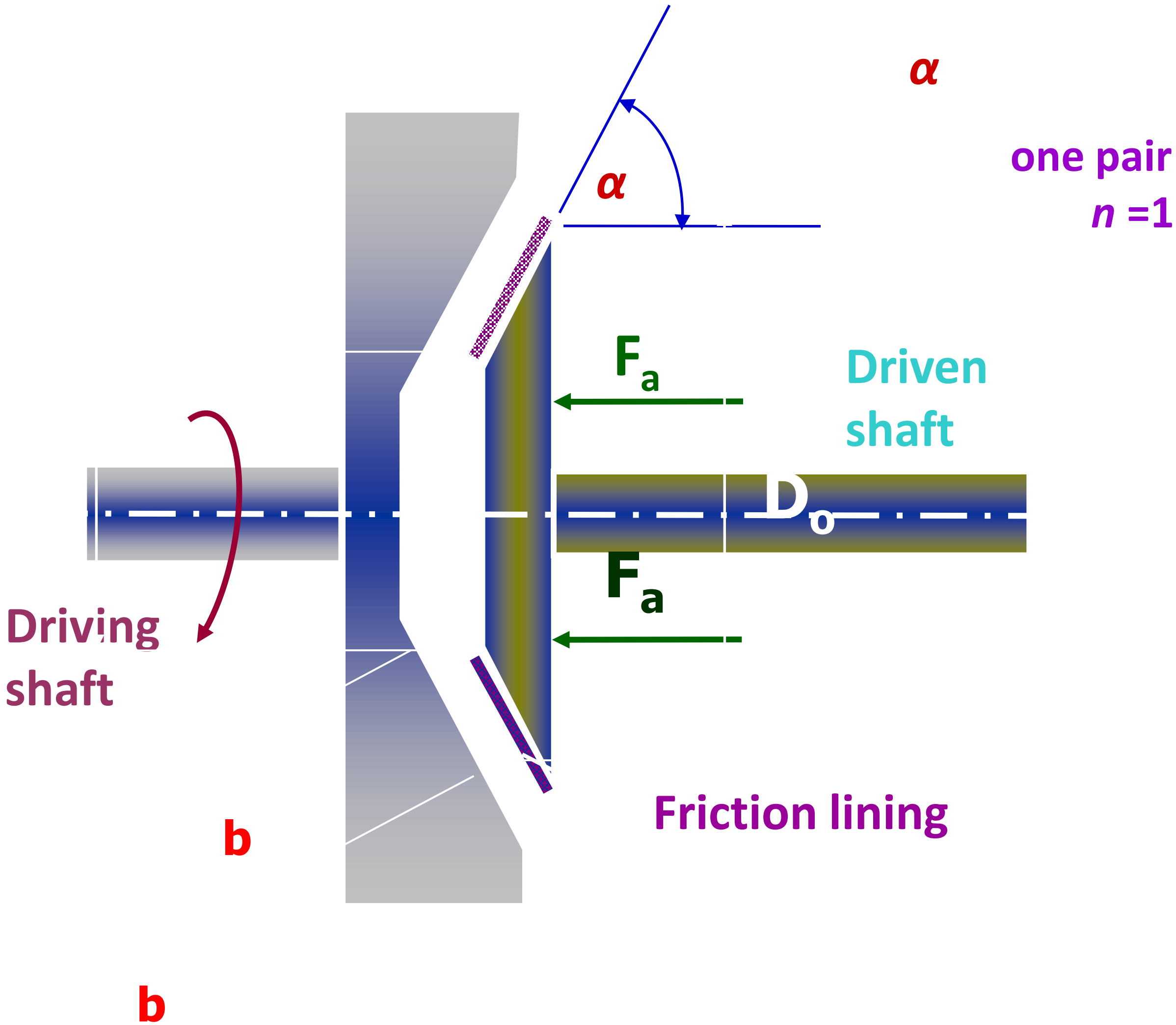
$$P = 2\pi NT/60\times1000 \qquad kW$$

N = speed in rpm

T = Torque transmitted

Cone clutch

Cone clutches are generally now only used in low peripheral speed applications although they were once common in automobiles and other combustion engine transmissions. They are usually now confined to very specialist transmissions in racing, rallying, or in extreme off-road vehicles, although they are common in power boats.


α
one pair
n =1
α
F_a
Driven
shaft
D_o
F_a
Driving
shaft
Friction lining
b
b

If the clutch is engaged when one member is stationary and the other is rotating, there is force resisting engagement, and

$$F_a = F_n\,(\sin\alpha + f\cos\alpha)$$

- The width of a cone face may be determined using the relation

  $$\mathbf{F_n = \pi D_m b p}$$

The dimensions of a cone clutch can be taken empirically as

$D_m/b = 4.5$ to 8

$D_m = 5D$ to $10D$ where D =shaft diameter

Design of cone clutches

- The torque transmission capacity of the cone clutch is,

$$T = F_b \, D_m / 2$$

$$T = \tfrac{1}{2} f \, F_a \, D_m / \mathrm{Sin}\,\alpha$$ where

D_m = mean diameter= $(D_o + D_i)/2$.

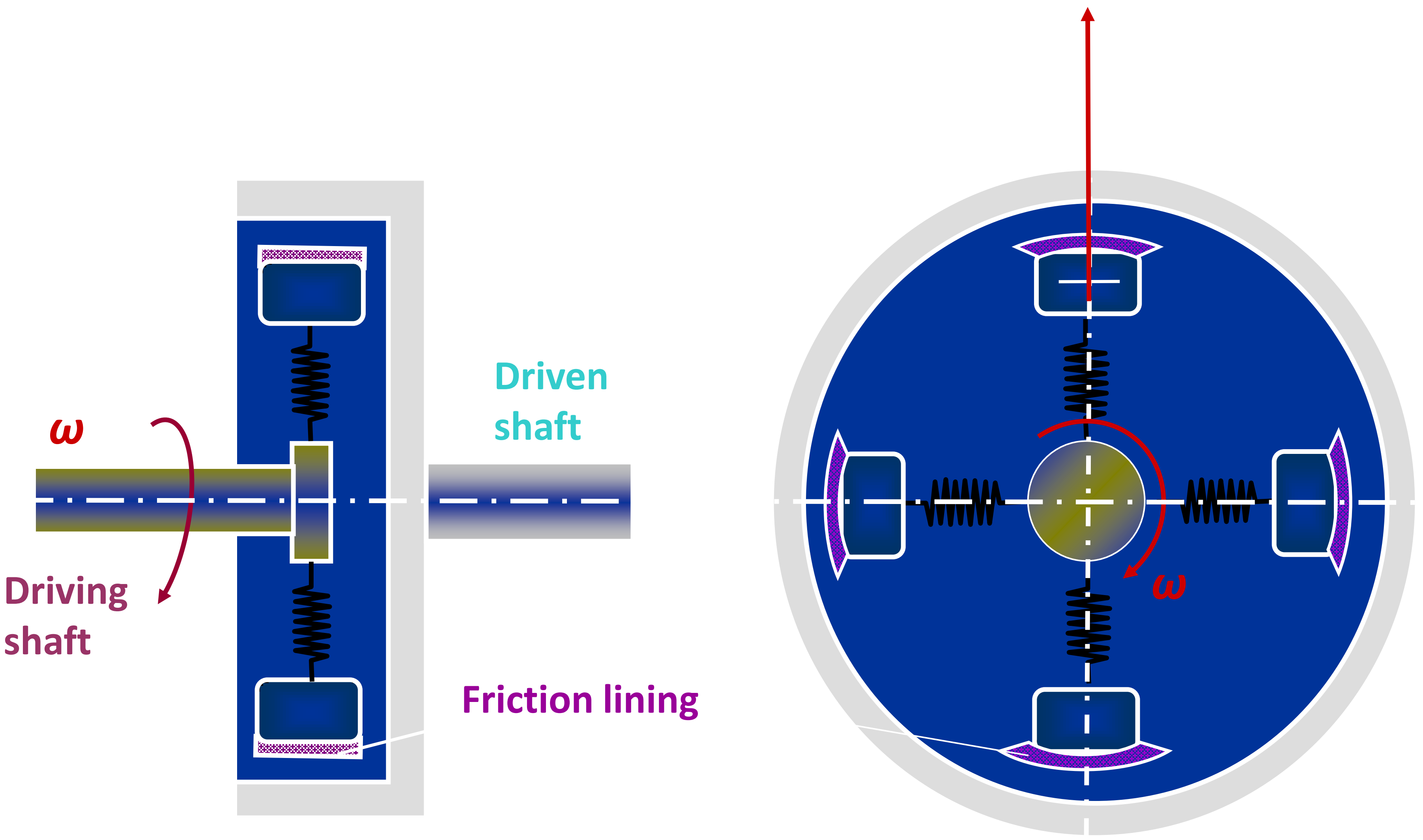
α = semi apex angle of the cone

Chapter#14 Clutches

- A centrifugal clutch works on the principle of centrifugal force.
- The clutch's purpose is to disengage when the engine is idling so that the chain does not move .
- These clutches are particularly useful in internal combustion engines, which can not be started under load.

- The clutch consists of three parts:
- **An outer drum that turns freely - This drum includes a sprocket that engages the chain. When the drum turns, the chain turns.**
- **A center shaft attached directly to the engine's crankshaft - If the engine is turning, so is the shaft.**
- **A pair of cylindrical clutch weights attached to the center shaft, along with a spring that keeps them retracted against the shaft**

$$F_c = mr\omega^2$$



There are several advantages to a centrifugal clutch

- It is automatic. (In a car with a manual transmission, you need a clutch pedal. A centrifugal clutch doesn't.)
- It slips automatically to avoid stalling the engine. (In a car, the driver must slip the clutch.)
- Once the engine is spinning fast enough, there is no slip in the clutch.
- It lasts forever.

Centrifugal clutch

The torque transmitting capacity of the clutch is given by

$$\mathbf{T = n\ F_f\ R}$$

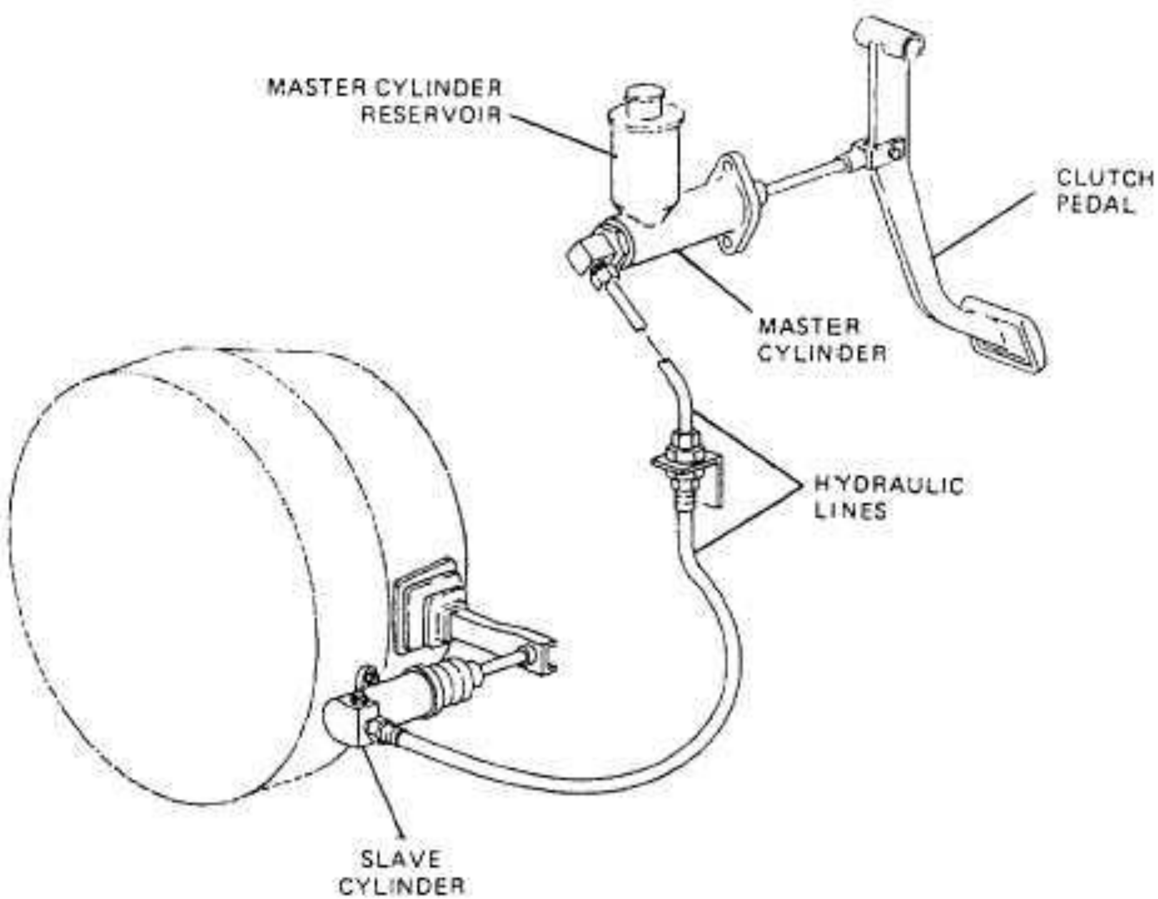
where R =radius of the drum,

n= number of shoes

Hydraulic clutches

- The term 'hydraulic clutch' denotes a clutch which utilizes the forces of gravity of a liquid for the transmission of power.
- Depressing the clutch pedal creates pressure in the clutch master cylinder, actuating the slave cylinder which, in turn, moves the release arm and disengages the clutch
- Hydraulic types of clutch operating systems are found in heavy construction equipments where extreme pressures are required for clutch operation.

Hydraulic clutches



THANKS