Strength Of Materials

Subject Code: 27061

Semester: 6th

Department: Mechanical & Power

Presented by

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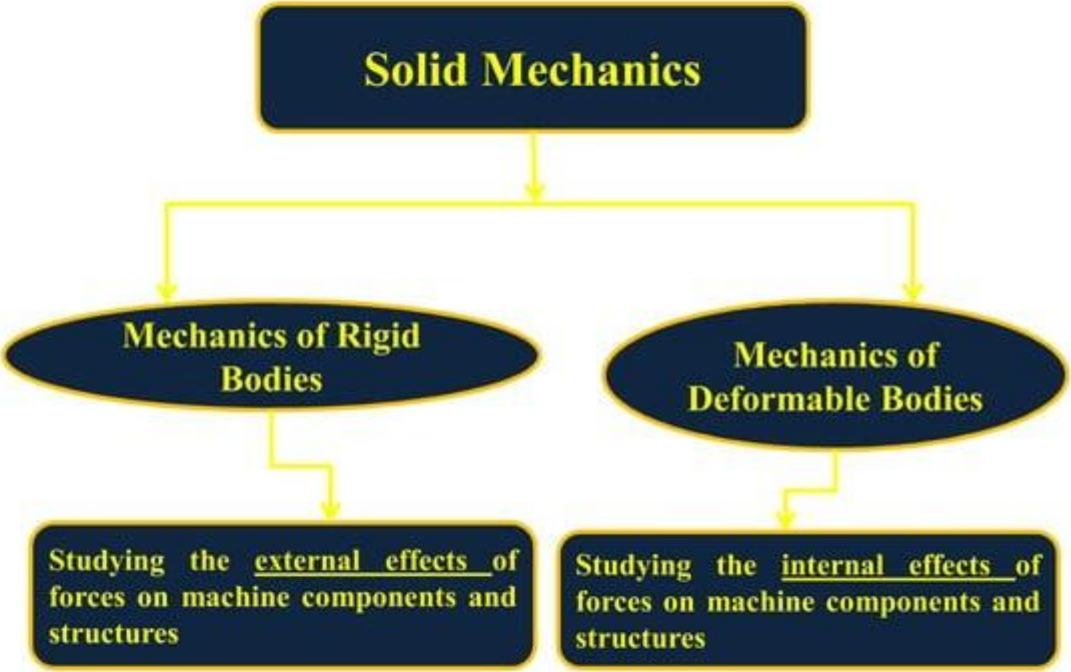
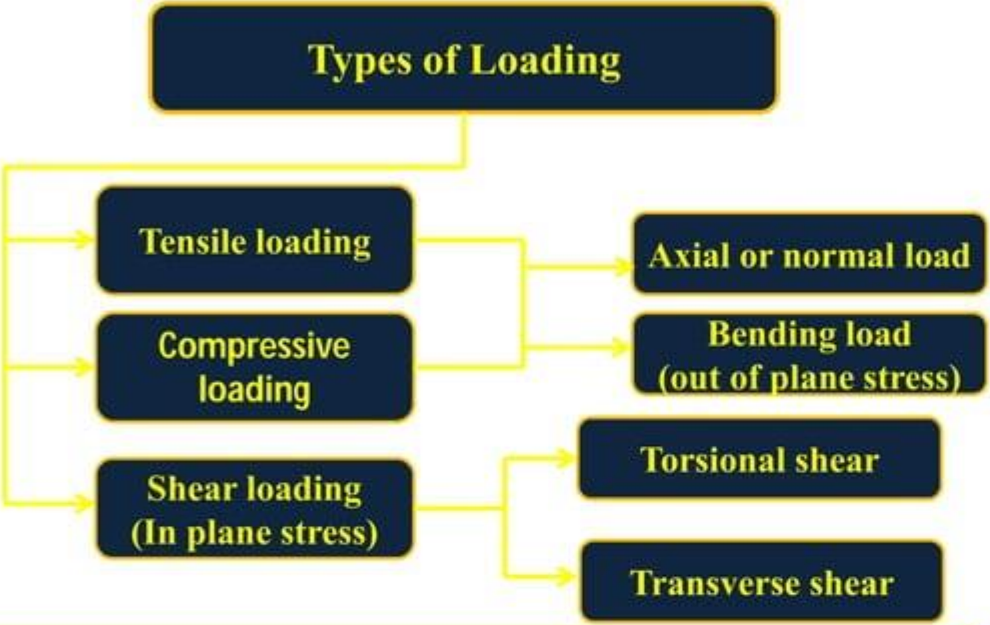
Workshop Super (Mechanical)

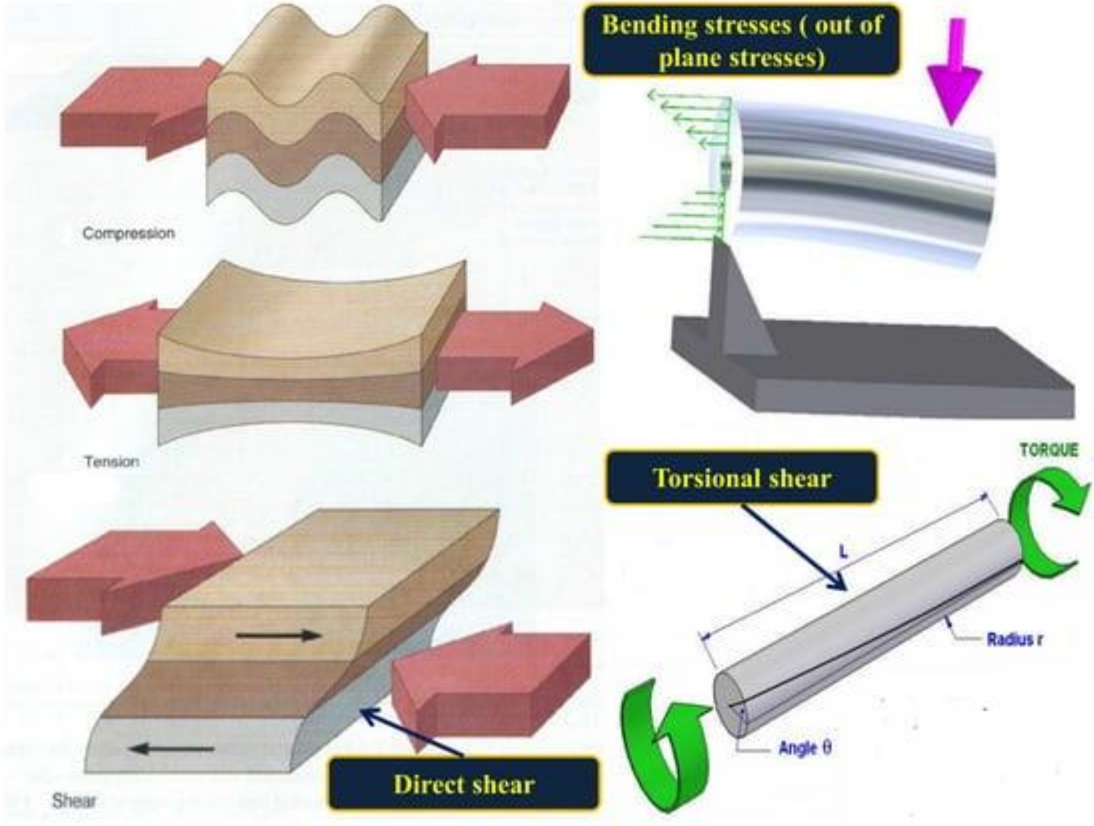
Mymensingh Polytechnic Institute


Solid Mechanics
Mechanics of Rigid Bodies
Mechanics of Deformable Bodies
Studying the external effects of forces on machine components and structures
Studying the internal effects of forces on machine components and structures

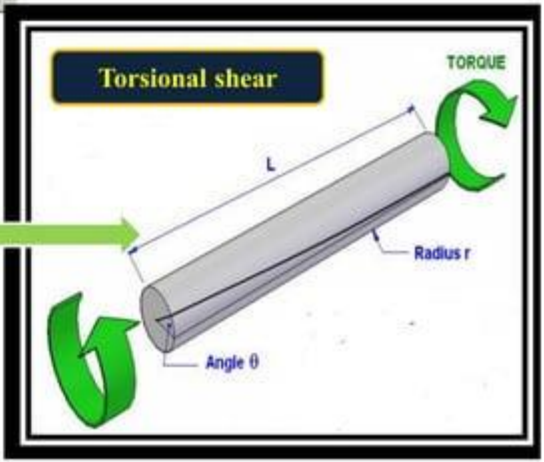


Normal loading : Tensile, compressive (out of plane)

Shear loading: tangential loading or parallel loading (in plane loading)


Compression
Tension
Shear
Direct shear
Bending stresses (out of plane stresses)
Torsional shear
TORQUE
L
Radius r
Angle θ

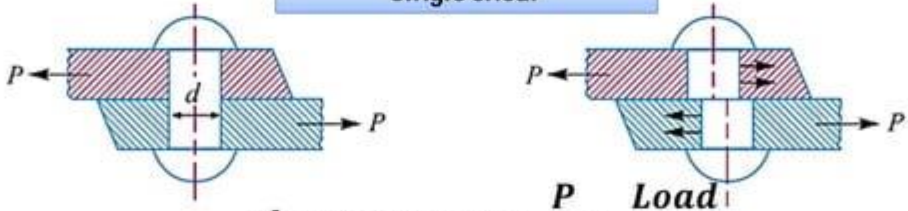
An effect that produces shifting of horizontal planes of a material



The twisting and distortion of a material's fibers in response to an applied load

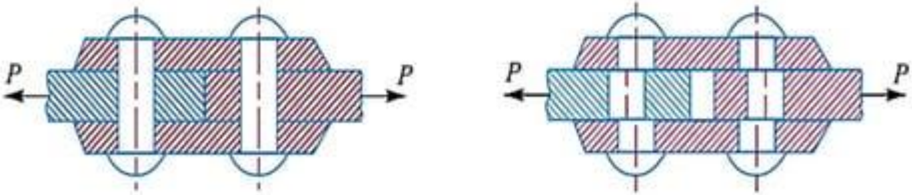
Direct Shear Stress - Examples

Single shear



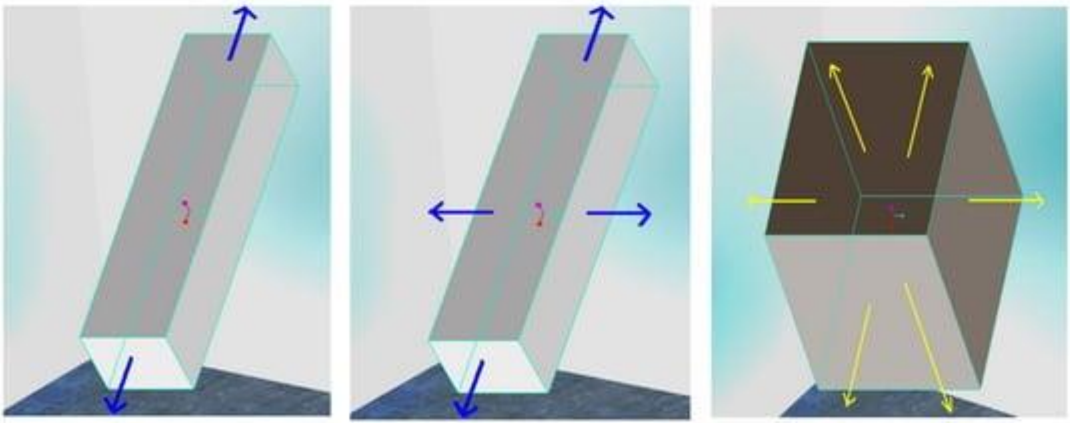
$$\textit{shear stress}\ \tau = \frac{P}{A} = \frac{Load}{Area}$$

Double shear



$$\textit{shear stress}\ \tau = \frac{P}{2A} = \frac{Load}{2(Area)}$$

Different types of Loadings



Uniaxial loading

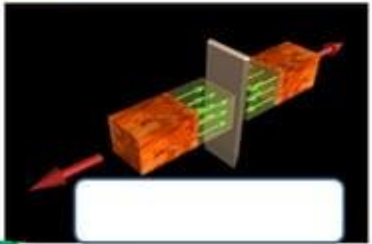
Biaxial loading

Triaxial loading

$$\text{Stress } [\sigma] = \frac{\textit{Applied load}}{\textit{Area of cross section}} \; N/mm^2$$

Types of stresses (plane stress, volumetric stress)

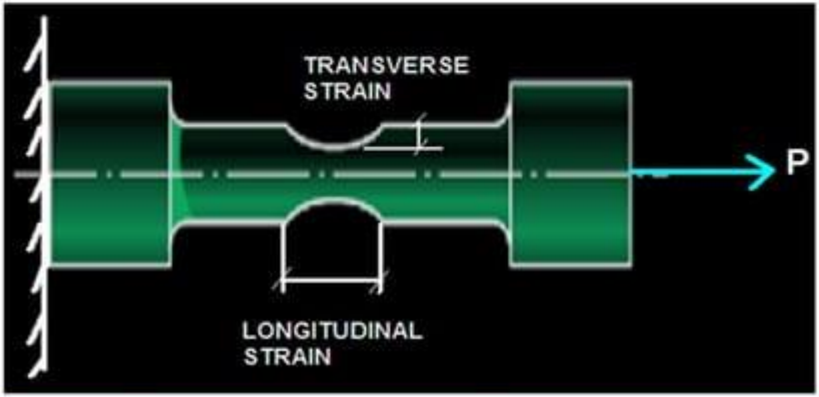
- **Normal stresses**
- **Shear stresses (or) tangential (or) parallel stresses [τ]**



$$\text{Strain } [\varepsilon] = \frac{\textit{change in length}}{\textit{original length}}$$

Types of strain (plane strain , volumetric strain)

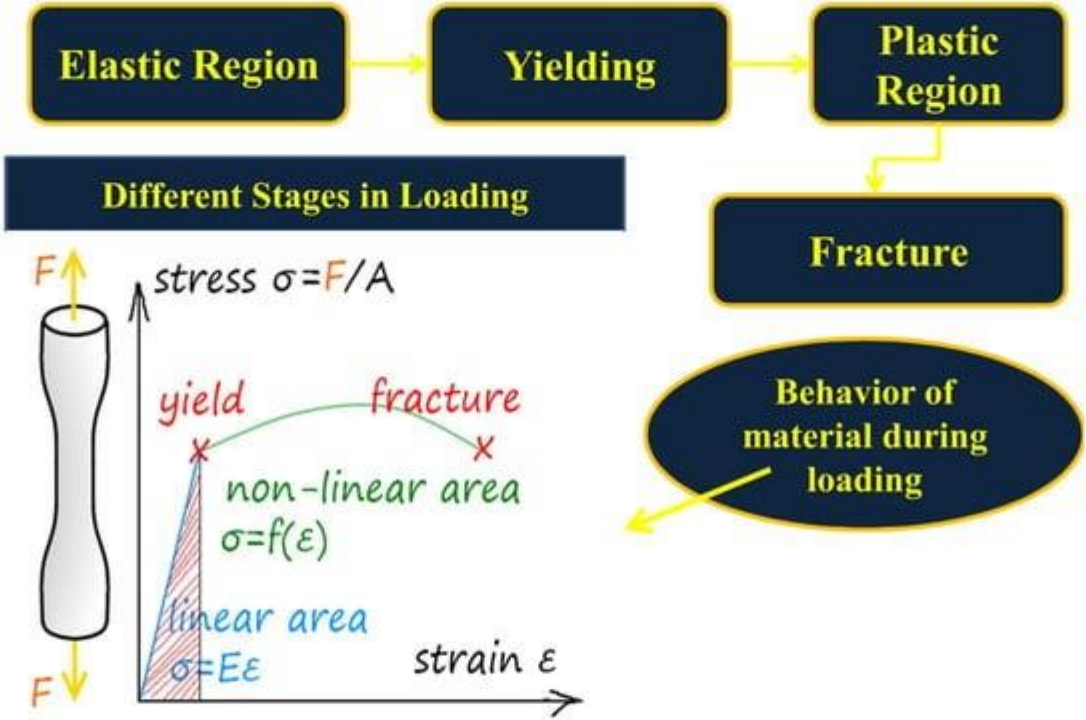
- **Longitudinal (or) linear strain**
- **Transverse (or) lateral strain**



Within the elastic limit, lateral strain is directly proportional to longitudinal strain.

$$\frac{\textit{lateral strain}}{\textit{linear strain}} = \text{constant}$$

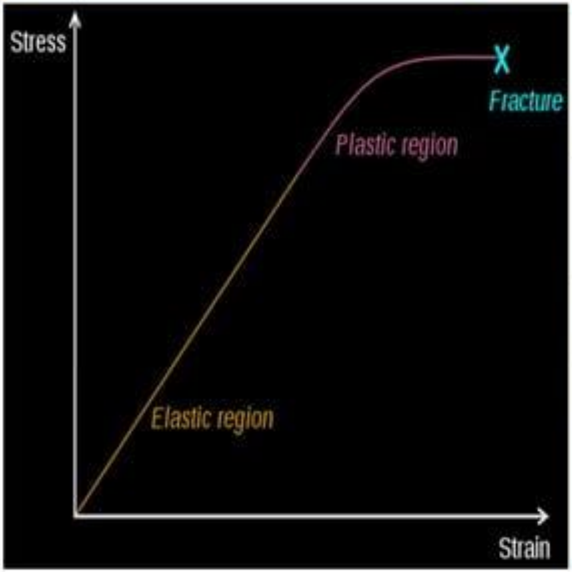
This constant is known as Poisson's ratio which is denoted by μ (or) 1/m.
Note: Linear strain = - μ × Lateral strain



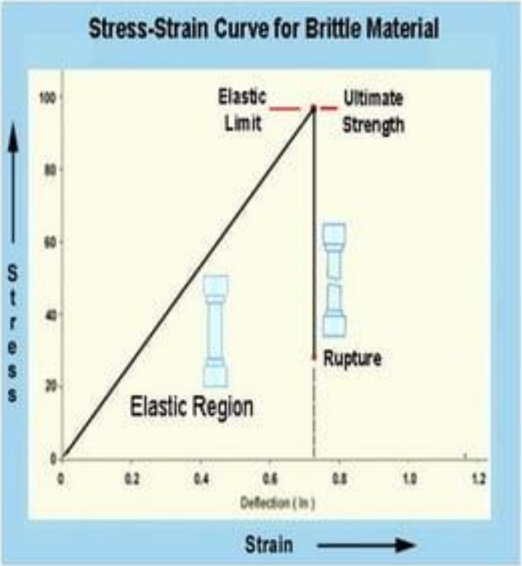
Elastic Region
Yielding
Plastic Region
Fracture
Different Stages in Loading
Behavior of material during loading
F
stress σ=F/A
yield
fracture
non-linear area
σ=f(ε)
linear area
σ=Eε
strain ε
F

Stress Strain Curve

Ductile materials



Brittle materials



Hooke's Law

Its for tensile, compressive and shear stresses

In 1678, Robert Hooke's found that for many structural materials, within certain limits, elongation of a bar is directly proportional to the tensile force acting on it. (Experimental finding)

Statement : Within the elastic limit, stress is directly proportional to strain.

$$Stress\ (\sigma)\ \alpha\ Strain\ (\varepsilon)$$

$$\sigma = E \in \quad \longrightarrow \quad \delta l = \frac{P\,l}{A\,E}$$

Where E = Young's Modulus or Modulus of Elasticity (MPa)

Modulus of Rigidity [or] Shear Modulus

Statement : Within the elastic limit, shear stress is directly proportional to shear strain.

$$shear\ Stress\ (\tau)\ \alpha\ shear Strain\ (\phi)$$

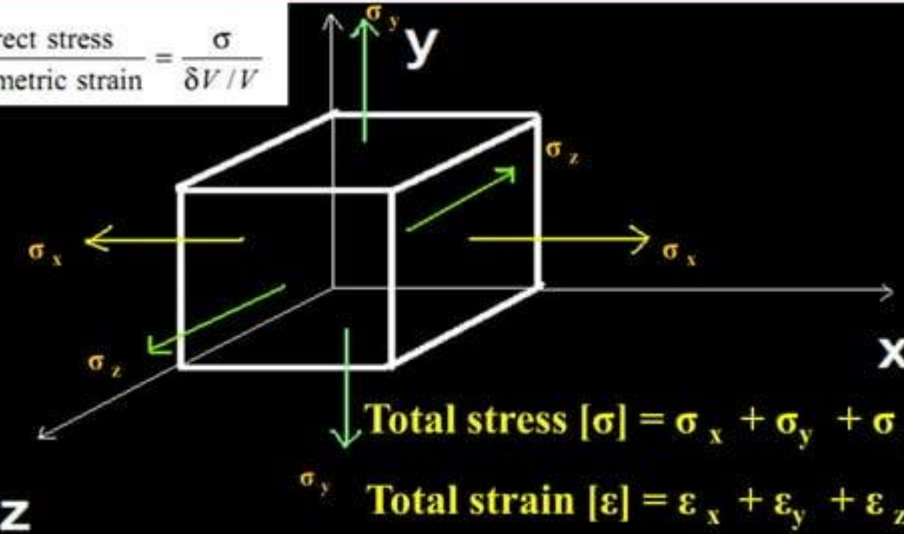
$$\tau = G\,\phi$$

Where G = Shear Modulus or Modulus of Rigidity (MPa)

Bulk Modulus

When a body is subjected to three mutually perpendicular stresses, of equal intensity, then the ratio of the direct stress to the corresponding volumetric strain is known as bulk modulus.

$$K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{\sigma}{\delta V / V}$$



Total stress $[\sigma] = \sigma_x + \sigma_y + \sigma_z$

Total strain $[\varepsilon] = \varepsilon_x + \varepsilon_y + \varepsilon_z$

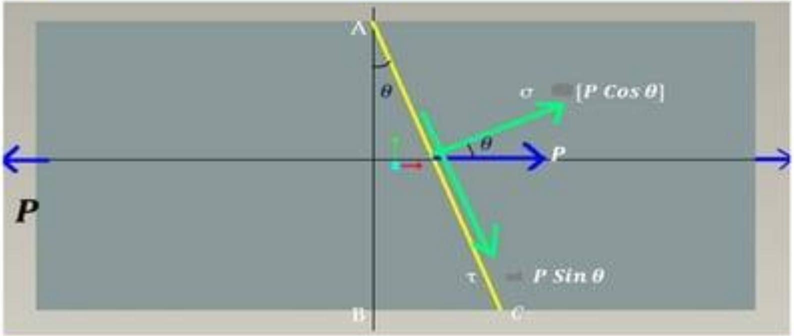
Analysis of Stresses in varying cross section bar

Principle of Superposition:

When a number of loads are acting on a body, the resulting strain will be the algebraic sum of strains caused by individual loads.

$$dL = \frac{P}{E}\left[\frac{L1}{A1} + \frac{L2}{A2} + \frac{L3}{A3}\right]$$

Stress in an Inclined Plane



- The plane perpendicular to the line of action of the load is a principal plane. [Because, It is having the maximum stress value and shear stress in this plane is zero.]
- The plane which is at an angle of 90° will have no normal and tangential stress.

$$\tau = 0.5\,\sigma$$

In triangle ABC, $\cos\theta = AB / AC$

$AC = AB / \cos\theta$

Stress along the inclined plane (shear stress)

$$\tau = P \sin\theta \cos\theta / AB$$

$$= \frac{P}{AB} \sin 2\theta / 2$$

$$\tau = \frac{\sigma}{2} \sin 2\theta$$

2θ is maximum when $\theta = 45°$

Therefore, $\tau = 0.5\,\sigma$

Max. shear stress = ½ Normal stress

Note to Remember:

- When a component is subjected to a tensile loading, then both tensile and shear stresses will be induced. Similarly, when it is subjected to a compressive loading, then both compressive and shear stresses induced.

- At one particular plane the normal stress (either tensile or compressive) will be maximum and the shear stress in that plane will be zero. That plane is known as principal plane. That particular magnitude of normal stress is known as principal stress.

- The magnitude of maximum principal stress will be higher when compare with the loads acting.

Elastic Constants (E, G, K and μ)

$$E = 3K[1 - 2\mu]$$

$$E = \frac{9KG}{3K + G}$$

$$G = \frac{E}{2[1 + \mu]}$$

Where
E = Young's Modulus
K = Bulk Modulus
G [or] C = Shear Modulus
μ = Poisson's Ratio

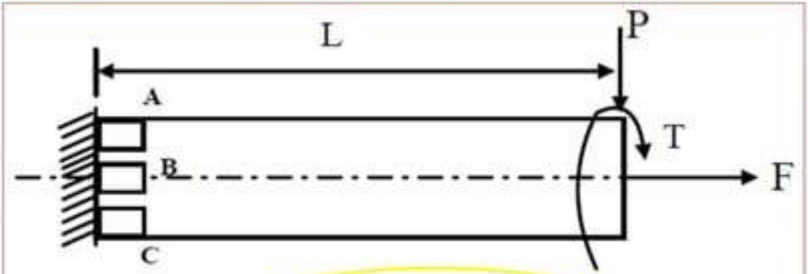
For pure shear,

Total tensile strain $= \frac{\tau}{E}[1 + \mu]$

Total tensile strain = ½ × Total shear strain

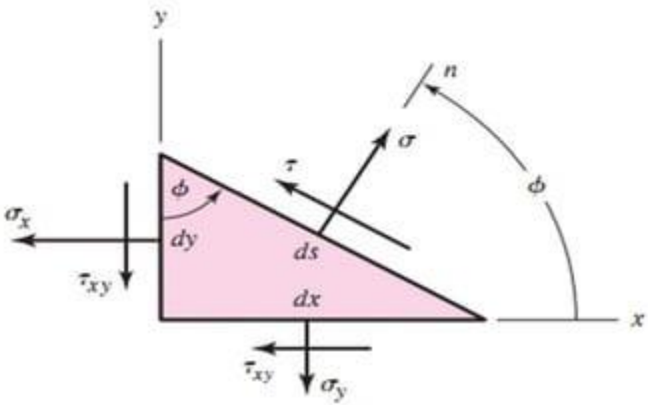
Principal Stresses

When a combination of axial, bending and shear loading acts in a machine member , then identifying the plane of maximum normal stress is difficult . Such a plane is known as principal plane and the stresses induced in a principal plane is known as principal stresses. Principal stress is an equivalent of all stresses acting in a member.



Combined loading

Biaxial and shear loading



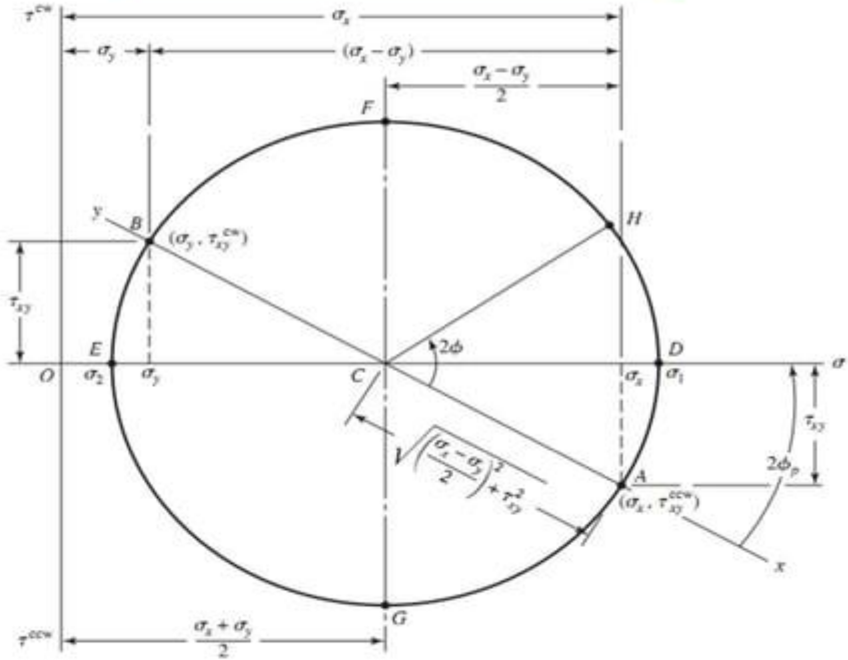
Max principal normal stresses

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Max principal shear stresses

$$\tau_1, \tau_2 = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Mohr's Circle Diagram



Thermal Stresses

When a body is subjected to heating or cooling, it will try to expand or contract. If this expansion or contraction is restricted by some external FORCES, then thermal stresses will be induced.

Let l = Original length of the body,

t = Rise or fall of temperature, and

α = Coefficient of thermal expansion,

$\therefore$ Increase or decrease in length,

$$\delta l = l.\alpha.t$$

If the ends of the body are fixed to rigid supports, so that its expansion is prevented, then compressive strain induced in the body,

$$\varepsilon_c = \frac{\delta l}{l} = \frac{l.\alpha.t}{l} = \alpha.t$$

$\therefore$ Thermal stress, $\sigma_{th} = \varepsilon_c.E = \alpha.t.E$

Actual Thermal Stresses

Stress and strain when fixed end yields by an amount δ,

Then Actual expansion = $\alpha.T.L - \delta$

Actual strain = $[\frac{\alpha.T.L - \delta}{L}]$

Actual thermal stress = $[\frac{\alpha.T.L - \delta}{L}] \times E$

Mechanical properties of materials

Strength → ability to resist external forces

Stiffness → ability to resist deformation under stress

Elasticity → property to regain its original shape

Plasticity → property which retains the deformation produced under load after removing load

Ductility → property of a material to be drawn into wire form with using tensile force

Brittleness → property of breaking a material without any deformation

Mechanical Properties of Materials

Contd. . .

Malleability → property of a material to be rolled or hammered into thin sheets

Toughness → property to resist fracture under impact load

Machinability → property of a material to be cut

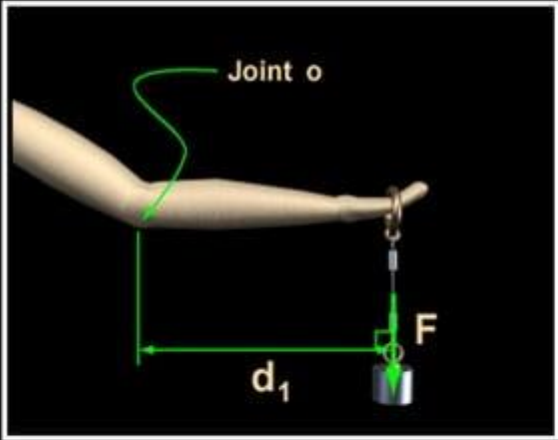
Resilience → property of a material to absorb energy

Creep → material undergoes slow and permanent deformation when subjected to constant stress with high temperature

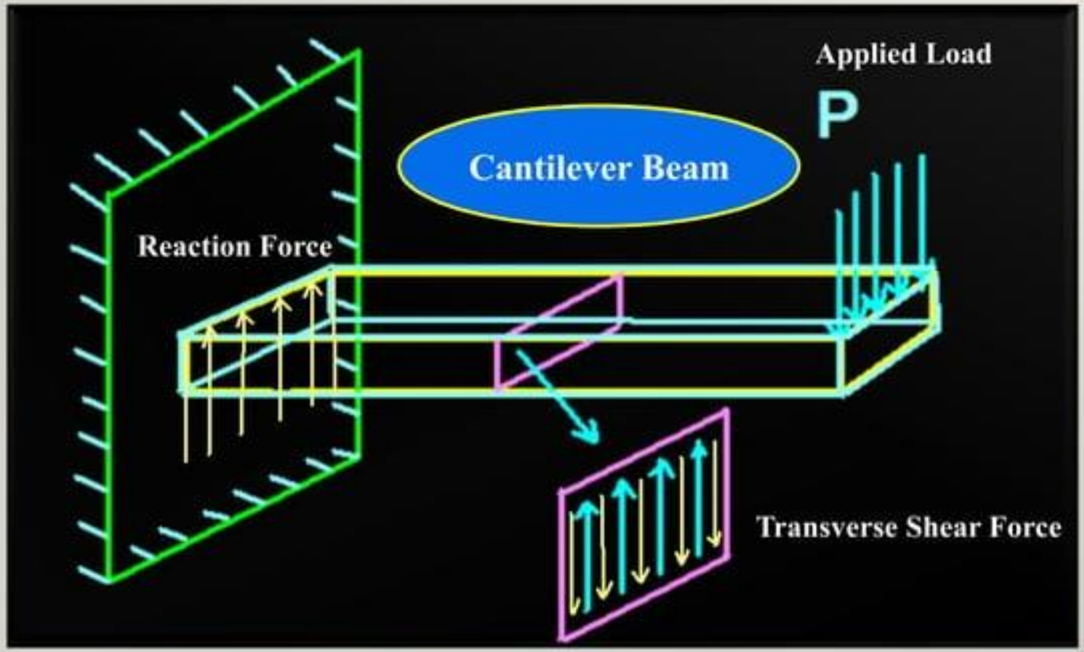
Fatigue → failure of material due to cyclic loading

Hardness → resistant to indentation, scratch

Bending Moment

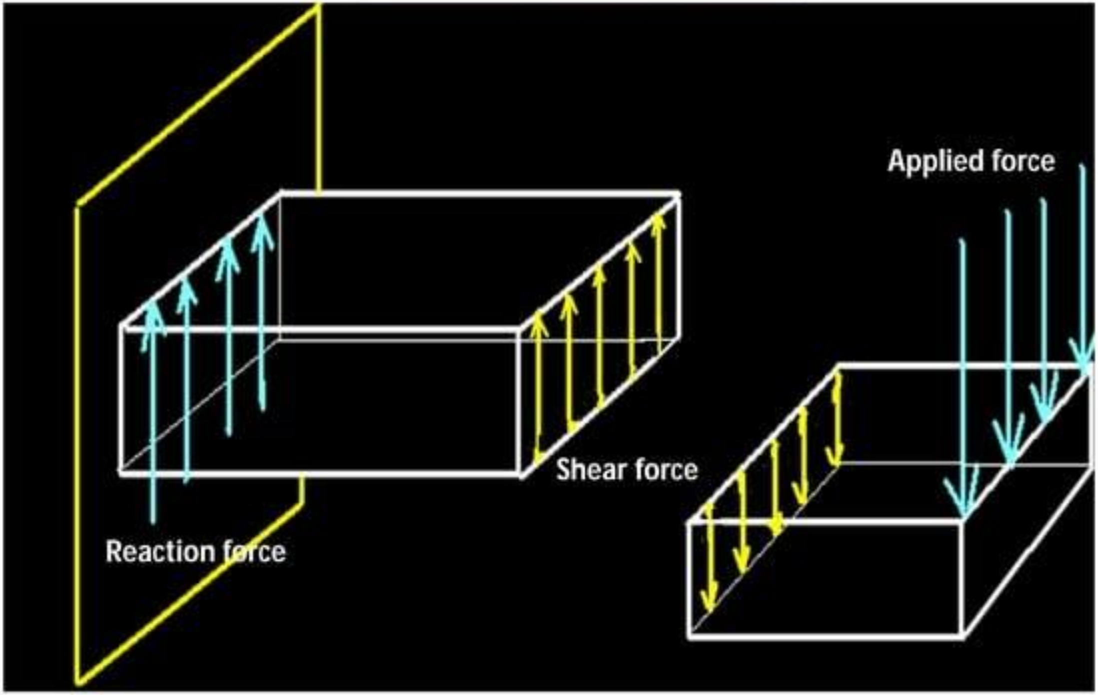


"Bending moment = Force × Perpendicular distance between the line of the action of the force and the point about which the moment produced"



"Vertical force in a section which is in transverse direction perpendicular to the axis of a beam is known as transverse shear force"

Shear Force (S.F)



Points to remember

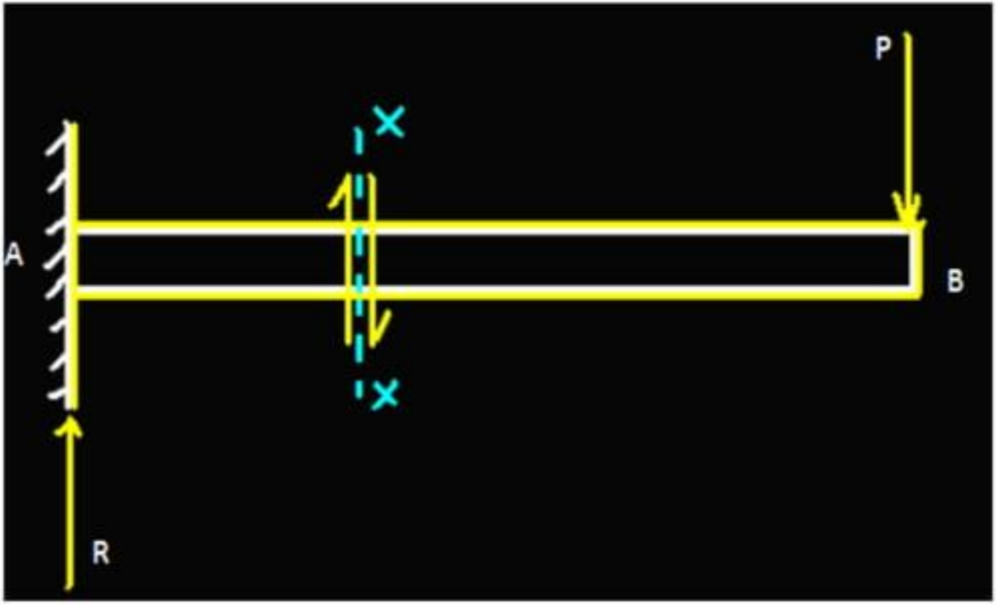
- For drawing shear force diagram, consider left or right portion of the section in a beam.

- Add the forces (including reaction) normal to the beam on one of the portion. If the right portion is chosen, a force on the right portion acting downwards is positive while a force acting upwards is negative and vice versa for left portion.

- Positive value of shear force are plotted above the base line.

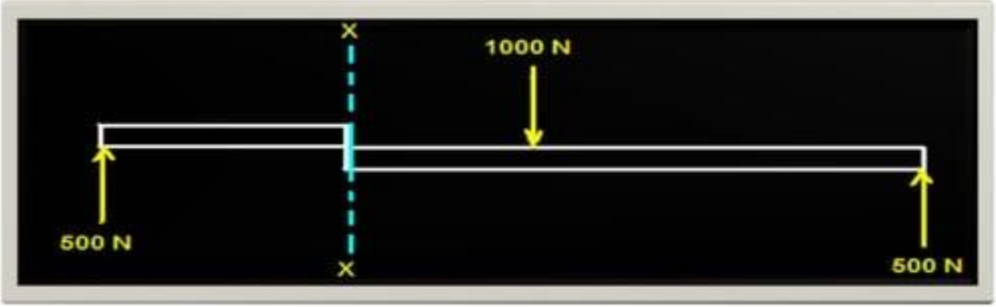
- S.F. Diagram will increase or decrease suddenly by a vertical straight line at a section where there is point vertical load.

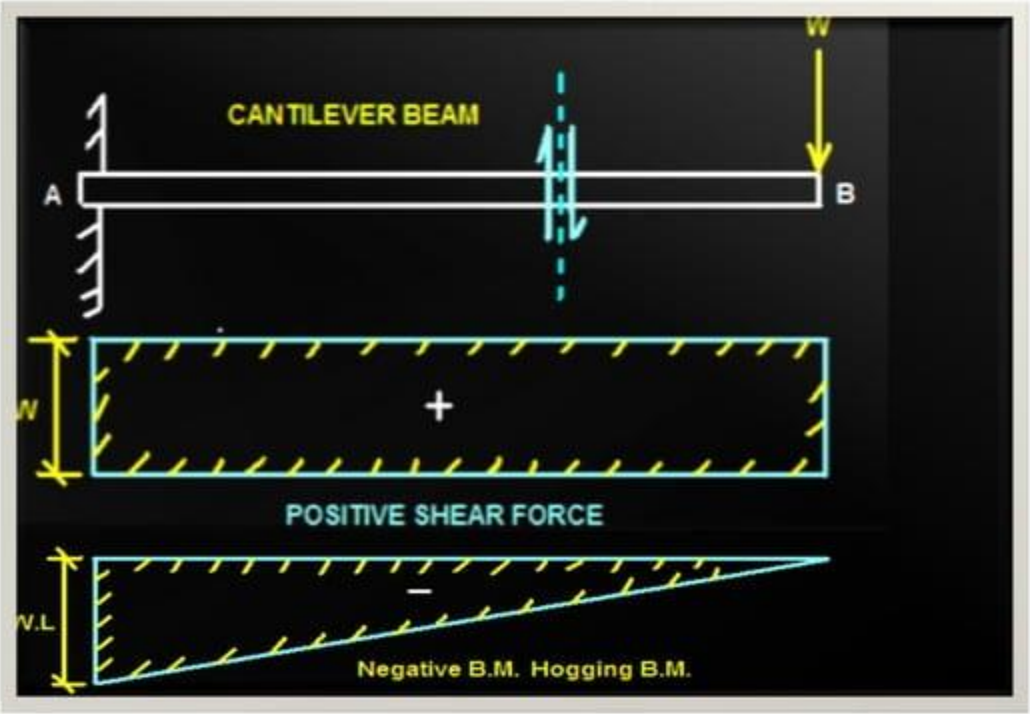
- Shear force between two vertical loads will be constant.



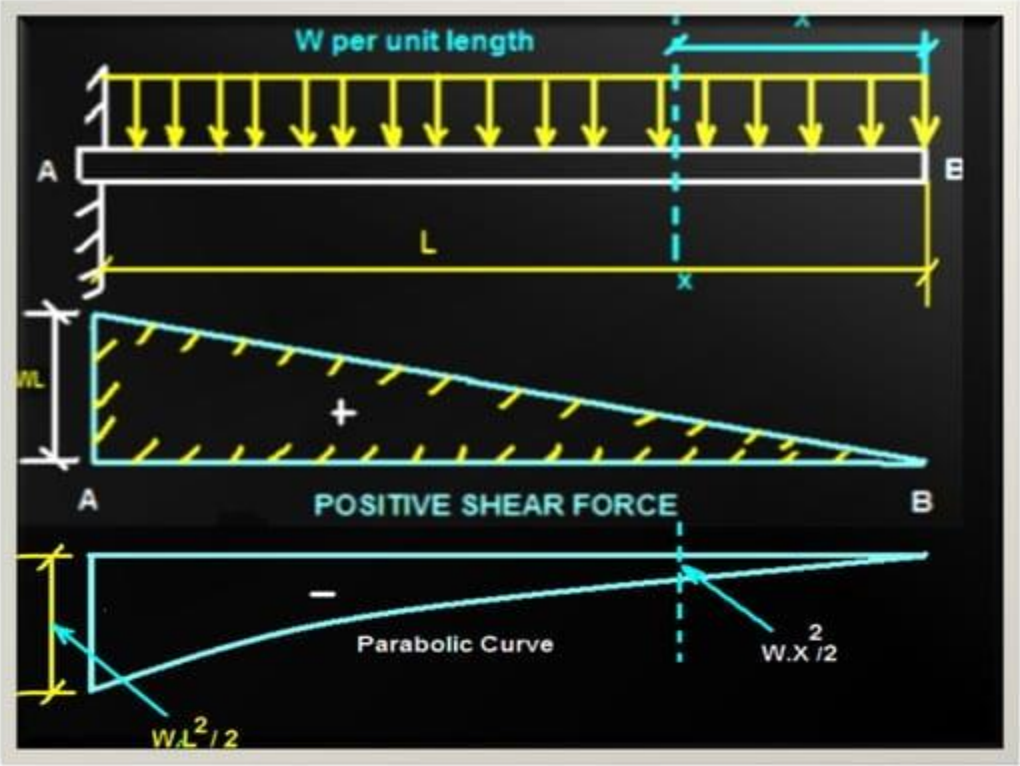
"The resultant force acting on any one portion (left or right) normal to the axis of the beam is called shear force at the section X-X. This shear force is known as transverse shear force"

Simply Supported Beam carrying 1000 N at its midpoint. The reactions at the supports will be equal to $R_A = R_B = 500$ N

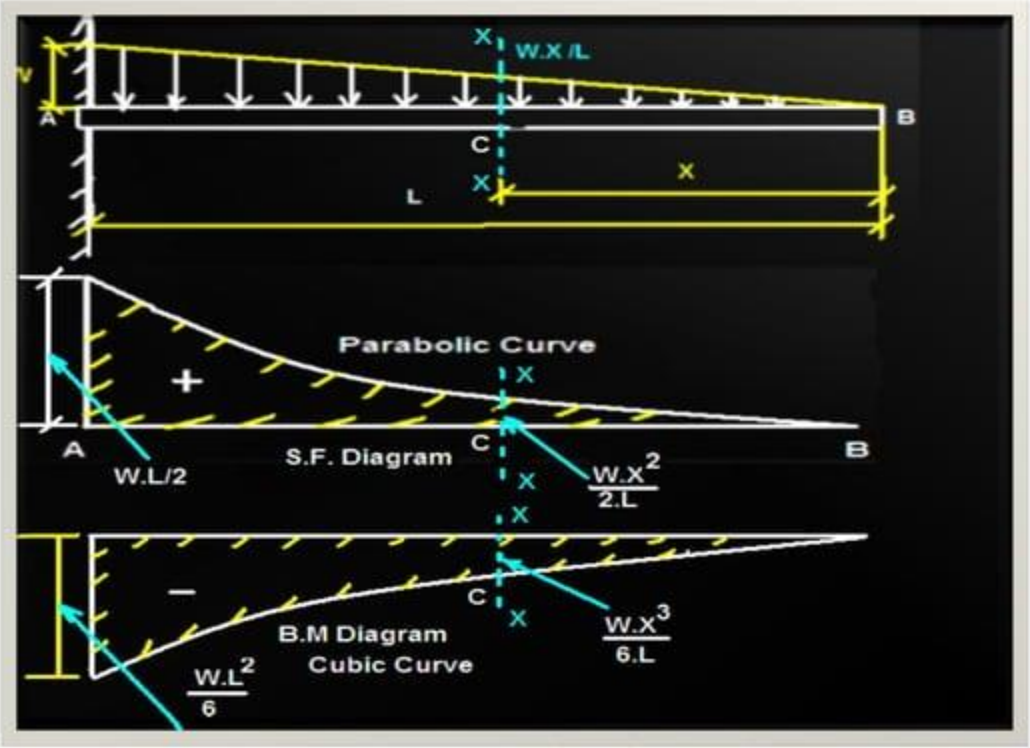




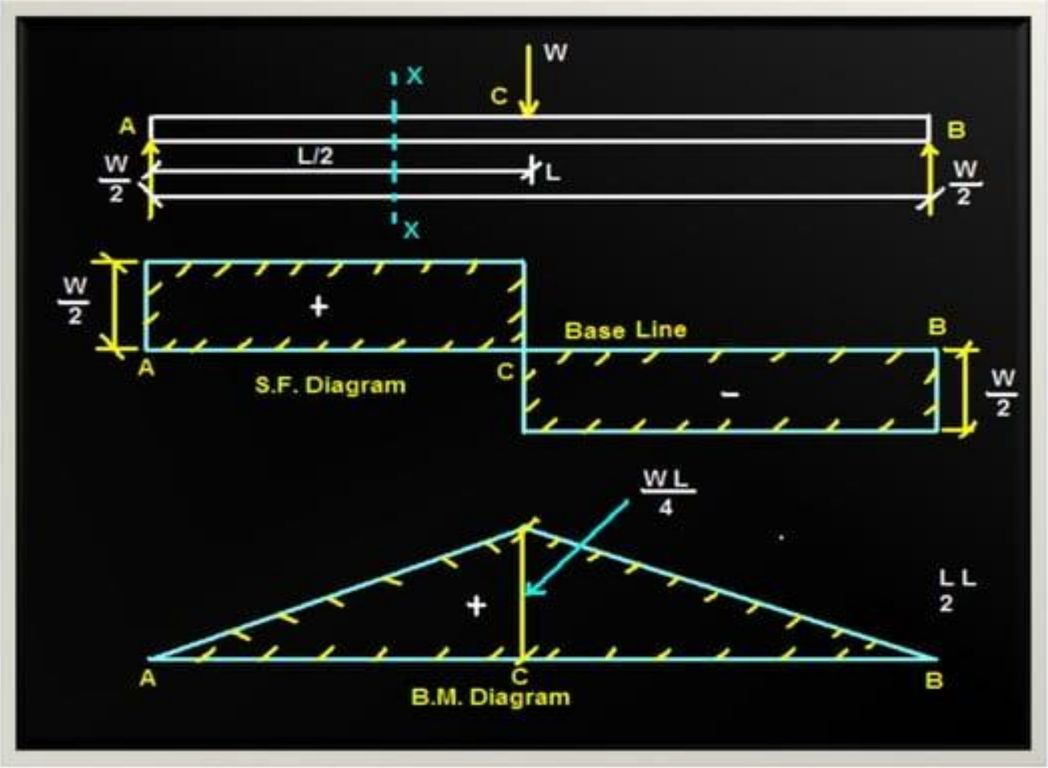
W
CANTILEVER BEAM
A
B
+
POSITIVE SHEAR FORCE
–
Negative B.M. Hogging B.M.

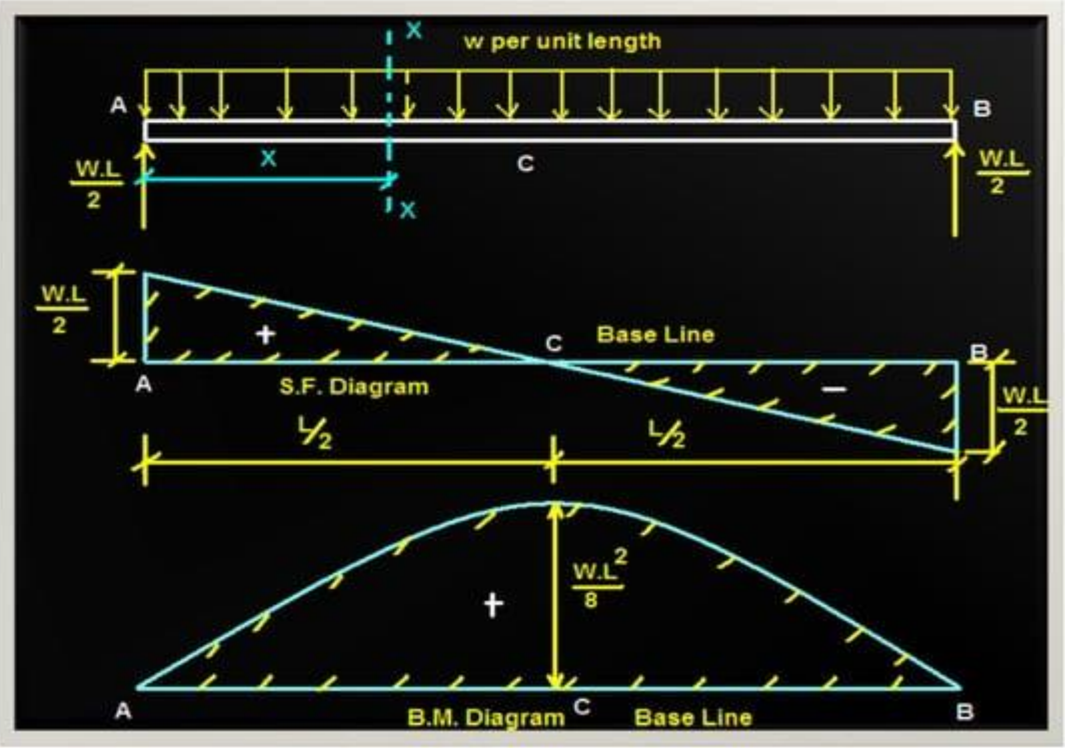


W per unit length
x
A
B
L
x
WL
+
A
POSITIVE SHEAR FORCE
B
−
Parabolic Curve
W.X²/2
W.L²/2

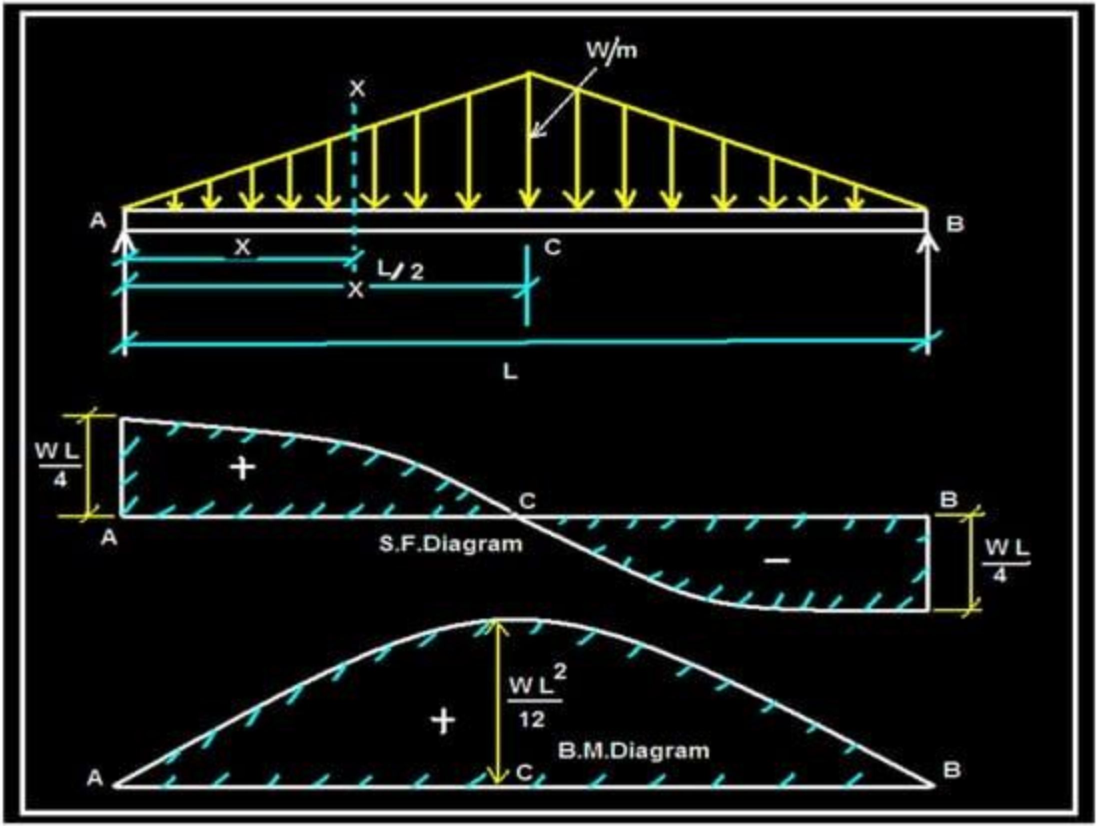


X
W.X /L
A
B
C
X
X
L
Parabolic Curve
X
+
A
C
B
S.F. Diagram
W.L/2
$\frac{W.X^2}{2.L}$
X
X
−
C
X
B.M Diagram
Cubic Curve
$\frac{W.X^3}{6.L}$
$\frac{W.L^2}{6}$

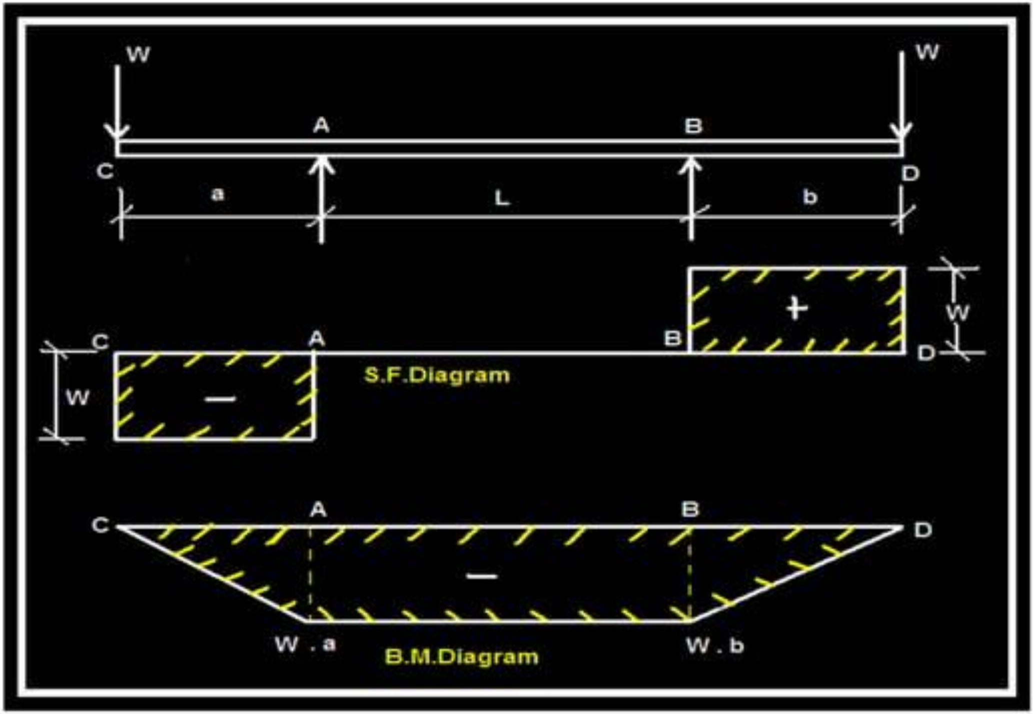

W
X
C
A
B
L/2
L
W/2
W/2
X
W/2
+
Base Line
B
A
C
S.F. Diagram
-
W/2
WL/4
+
LL
2
A
C
B
B.M. Diagram



w per unit length
X
A
B
W.L
2
X
C
W.L
2
X
W.L
2
+
Base Line
C
B
A
S.F. Diagram
–
W.L
2
L/2
L/2
W.L²
8
+
A
B.M. Diagram
C
Base Line
B


W/m
X
A
B
X
C
L/2
X
L
WL/4
+
C
B
A
S.F.Diagram
−
WL/4
WL²/12
+
B.M.Diagram
A
C
B

Pure Bending



Assumptions in the Evaluation of Bending stress

The beam is subjected to pure bending. This means that the shear force is zero, and that no torsion or axial loads are present.

The material is isotropic and homogeneous.

The material obeys Hooke's law.

The beam is initially straight with a cross section that is constant throughout the beam length.

The beam has an axis of symmetry in the plane of bending.

The proportions of the beam are such that it would fail by bending rather than by crushing, wrinkling, or sidewise buckling.

Plane cross sections of the beam remain plane during bending.

Thus Bending Equation is

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where,

M = Moment due to resistance

I = Moment of inertia of the section about the neutral axis

E = Young's modulus

R = Radius of curvature of neutral axis

σ = Bending stress

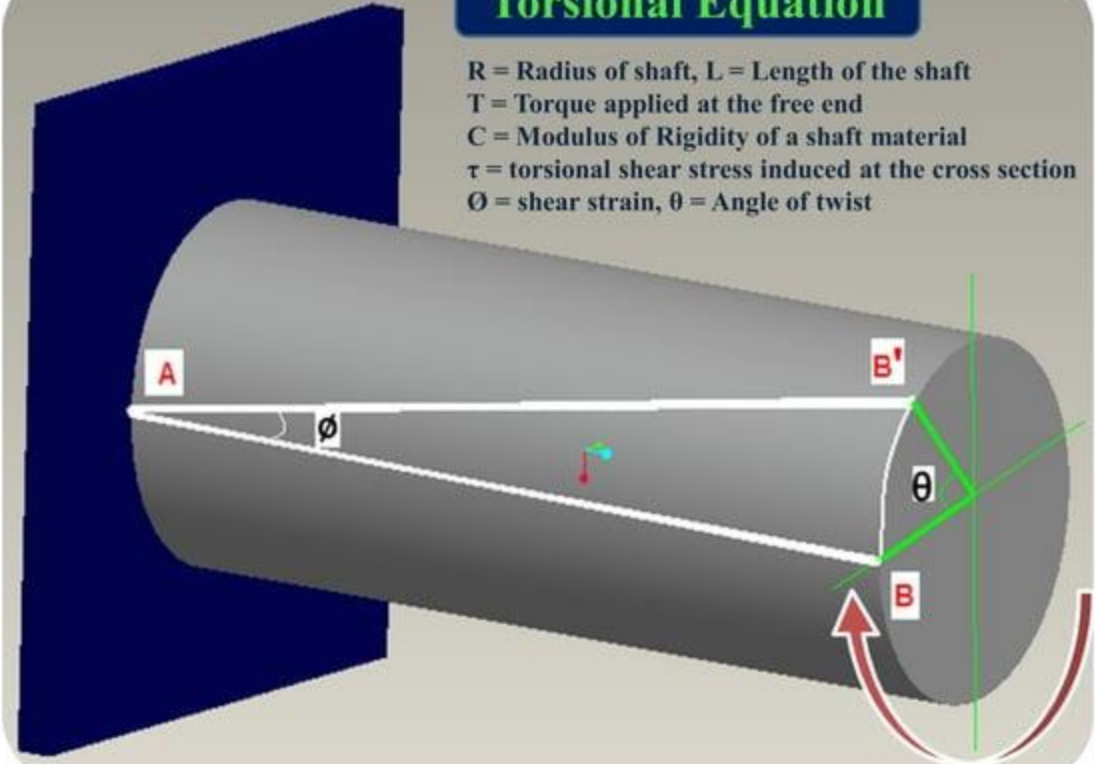
y = distance from the neutral axis to the extreme fiber of the cross section.

$$\text{Section modulus } Z = \frac{I}{y}$$

It is usually quoted for all standard sections. The strength of the beam mainly depends on the section modulus.

Torsional Equation

R = Radius of shaft, L = Length of the shaft
T = Torque applied at the free end
C = Modulus of Rigidity of a shaft material
τ = torsional shear stress induced at the cross section
Ø = shear strain, θ = Angle of twist



Due to twisting moment applied at the end of shaft, it will rotate clockwise and every cross section will be subjected to torsional shear stresses.

Therefore,

$$\text{Shear strain at the outer surface} = \text{Distortion per unit length} = \frac{BB''}{L} = \tan \phi \approx \phi$$

$$= \frac{R\,\theta}{L}$$

We know that,

$$\text{Modulus of Rigidity} = \frac{\text{shear stress at the outer surface}}{\text{shear strain at the outer surface}}$$

$$C \text{ or } G = \frac{\tau L}{R\,\theta} \quad \Longrightarrow \quad \frac{\tau}{R} = \frac{C\,\theta}{L}$$

Design for Bending

- When a shaft is subjected to pure rotation, then it has to be designed for bending stress which is induced due to bending moment caused by self weight of the shaft.

Example: Rotating shaft between two bearings.

Design for Bending & Twisting

- When a gear or pulley is mounted on a shaft by means of a key, then it has to be designed for bending stress (induced due to bending moment) and also torsional shear stress which is caused due to torque induced by the resistance offered by the key .

Example: gearbox shaft (splines)

Total Torque $[T] = \int_0^R dT = \int \frac{\tau}{R} \times 2\pi r^3\,(dr) = \int_0^R \frac{\tau}{R}\, r^2\, dA$

Therefore, $\frac{T}{J} = \frac{\tau}{R}$

Where,

J = Polar Moment of inertia

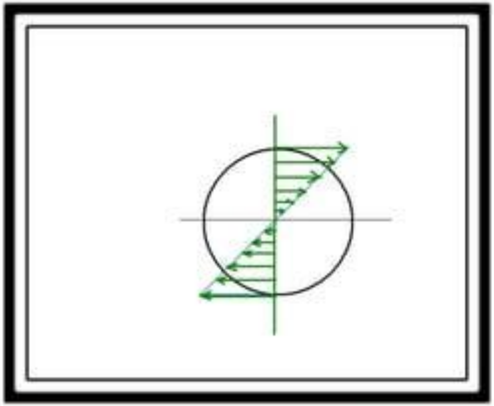
$$J = \int_0^R r^2\, dA = \frac{\pi}{32} D^4$$

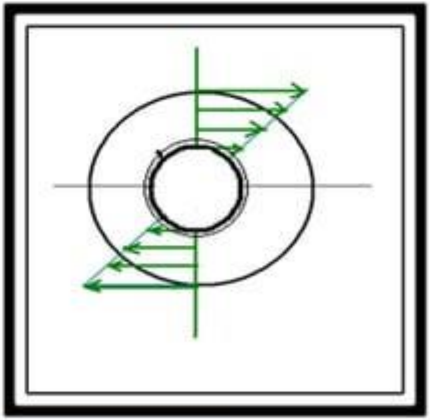
Therefore,

Torsional Equation,

$$\frac{T}{J} = \frac{\tau}{R} = \frac{C\,\theta}{L}$$

Shear stress distribution in solid & hollow shafts

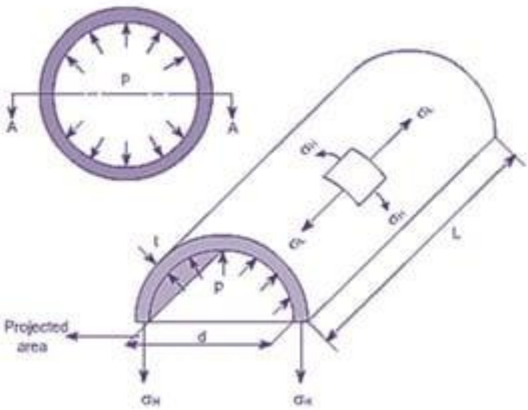




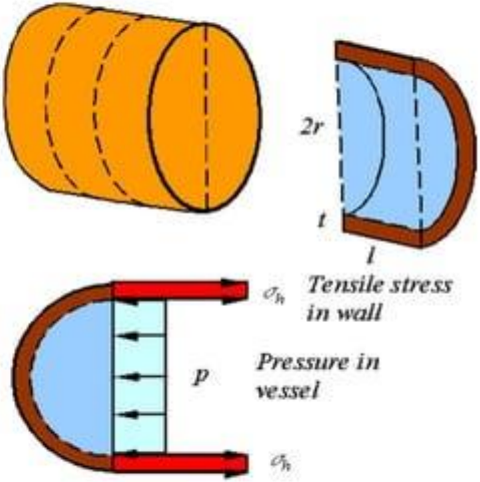
Torque transmitted by the solid shaft $T = \frac{\pi}{16} \tau D^3 \; N-m$

Torque transmitted by the hollow shaft $T = \frac{\pi}{16} \tau \; {Do^4 - Di^4}/{Do}$

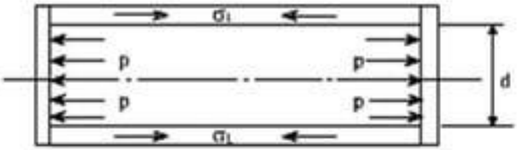
Power transmitted due to torque $P = \frac{2 \pi N T}{60}$ *Watts*

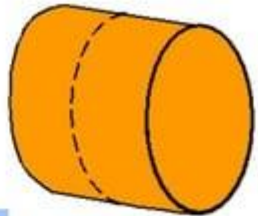


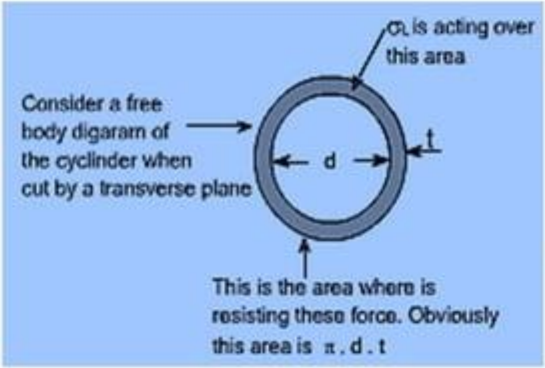
Hoop stress (or) Circumferential stress

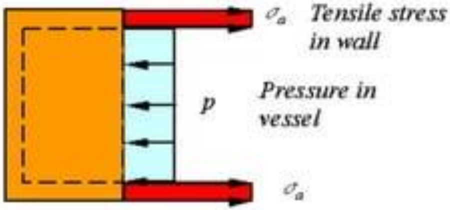


Circumferential or hoop Stress (σ_H) = (p .d)/ 2t

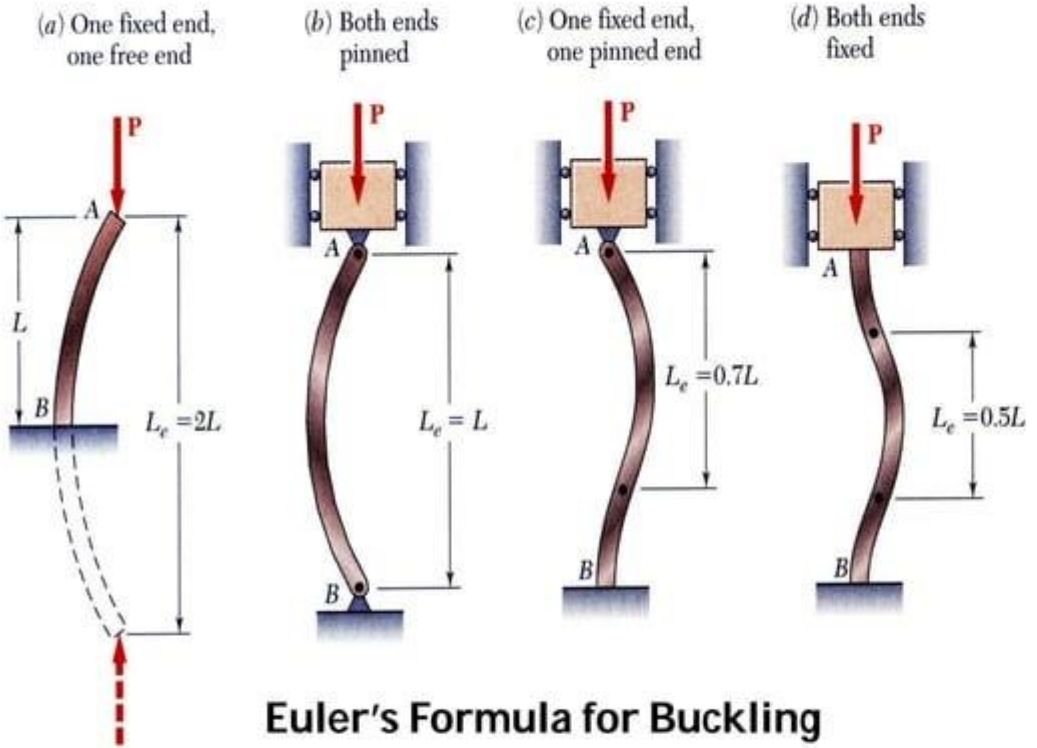






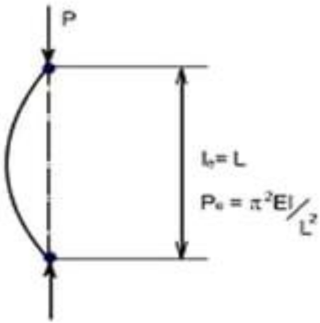


Longitudinal stress = (σ_L) = (p .d)/ 4t

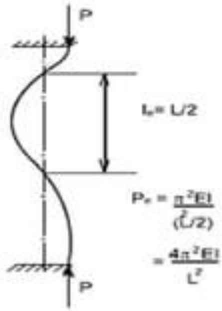


Euler's Formula for Buckling

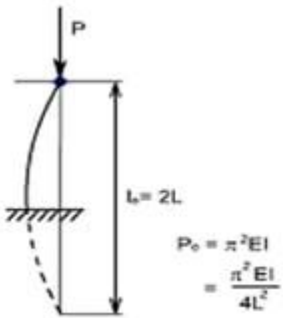
Both ends pinned



Both ends fixed



One end fixed, other free



One end fixed and other pinned